SSLS144 - Cylinder under internal pressure

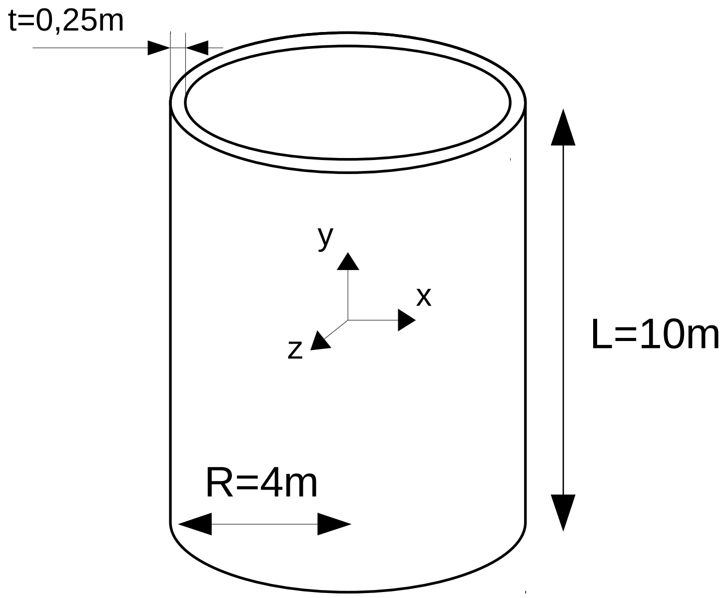
Summary:

This test valid modeling COQUE_AXIS (range by meshs SEG3). U is carried outN calculation static on a cylindrical tank under internal pressure. The model is axisymmetric. The distribution of pressure on the internal wall of the tank is not uniform. Displacements and efforts generalized obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry

A cylindr is consideredE of average radius $5m$, of thickness $t=0.25m$ and length $L=10m$. Selected geometrical dimensions make it possible to neglect effects edge free in $y=\pm L/2$ (into axisymmetric, L must check: $\frac{1}{2}L>3\sqrt{Rt}=3m$).



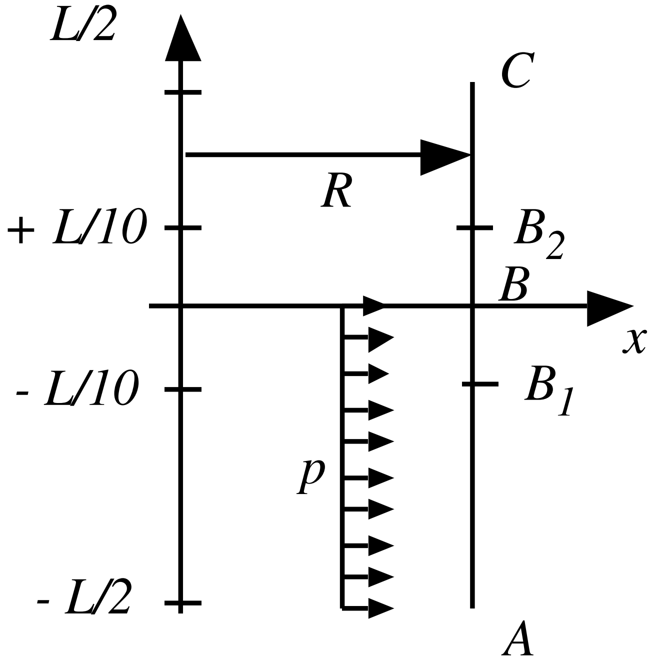
1.2 Properties of material

The isotropic elastic material is characterized by the properties Suivantes :

- $E=1Pa$
- $\nu=0.3$

1.3 Boundary conditions and loadings

The cylinder vertical is subjected to an internal pressure $p=1\,Pa$ constant on the part $y\leq 0$, and worthless on $y>0$. A vertical displacement is imposed $u_y=0$ at the point A .



2 Reference solution

2.1 Method of calculating

One places oneself in the theory of LOVE-KIRCHHOFF for the calculation of the analytical solution of this problem.

The arrow in the axisymmetric reference mark is written:

$$u_x=\begin{cases}\dfrac{P}{8\alpha^4 D}\left(2-e^{\alpha y}\cos(\alpha y)\right) & \forall\, y\leq 0\\ \dfrac{P}{8\alpha^4 D}e^{-\alpha y}\cos(\alpha y) & \forall\, y\geq 0\end{cases}$$

with $D=\dfrac{Et^3}{12(1-\nu^2)}$ and $4\alpha^4=\dfrac{Et}{DR^2}$.

Rotation is written:

$$\beta_s=\begin{cases}\dfrac{P}{8\alpha^3 D}e^{\alpha y}\left(\cos(\alpha y)-\sin(\alpha y)\right) & \forall\, y\leq 0\\ \dfrac{P}{8\alpha^3 D}e^{-\alpha y}\left(\cos(\alpha y)+\sin(\alpha y)\right) & \forall\, y\geq 0\end{cases}$$

Efforts generalizedS are:

$$N_{\theta\theta}=\frac{Et}{R}u_x(y)\ ,$$

$$M_{ss}=Du_x''(y)=\frac{p}{4\alpha^2}e^{-|y|}\sin(\alpha y)$$

The three-dimensional constraints are:

$$\sigma_{\theta\theta}=\frac{N_{\theta\theta}}{t}+12\frac{M_{\theta\theta}(x-R)}{t^3}\ ,$$

$$\sigma_{ss}=12\frac{M_{ss}(x-R)}{t^3}\ ,$$

that is to say :

$$\sigma_{\theta\theta}(y,x)=\begin{cases}\dfrac{pR}{t}\left(1-\dfrac{e^{\alpha y}}{2}\left(\cos(\alpha y)+2\nu\dfrac{R-x}{t}\sqrt{\dfrac{3}{1-\nu^2}}\sin(\alpha y)\right)\right) & \forall\, y\leq 0\\ \dfrac{pR}{t}\cdot\dfrac{e^{-\alpha y}}{2}\left(\cos(\alpha y)-2\nu\dfrac{R-x}{t}\sqrt{\dfrac{3}{1-\nu^2}}\sin(\alpha y)\right) & \forall\, y\geq 0\end{cases},$$

and:

$$\sigma_{ss}(y,x)=\begin{cases}\dfrac{pR}{t}\cdot\dfrac{x-R}{t}\sqrt{\dfrac{3}{1-\nu^2}}e^{\alpha y}\sin(\alpha y) & \forall\, y\leq 0\\ \dfrac{pR}{t}\cdot\dfrac{x-R}{t}\sqrt{\dfrac{3}{1-\nu^2}}e^{-\alpha y}\sin(\alpha y) & \forall\, y\geq 0\end{cases}.$$

2.2 Sizes and results of reference

The following values are tested :

- the arrow DX at the points A , B and C ,
- rotation DRZ , at the points A and B ,
- generalized effort NYY at the points B and B_1 ,
- generalized moment MXX at the point B_1

2.3 Bibliographical references

[1] S. ANDRIEUX - F. VOLDOIRE : Models of hulls. Applications in linear statics. School of Digital Summer CEA-EDF-INRIA of Analysis 1992.

2.4 Uncertainties on the solution

There is not

3 Modeling A

3.1 Characteristics of modeling

A modeling is used COQUE_AXIS. Models of COQUE_AXIS are usable as well for the thick plates (HENCKY-MINDLIN-REISSNER) as for the thin sections (KIRCHOFF-LOVE) thanks to an approach by penalization which makes it possible to neutralize or not the energy of shearing: it is the theory of HENCKY-MINDLIN-NAGHDI. In order to approach the solution of LOVE-KIRCHHOFF numerically, it is necessary to take a coefficient of sufficiently large shearing (A_CIS) to inhibit the transverse kinematics of shearing γ_s . The larger this coefficient is, the more the matrix of rigidity is almost singular thus source of digital instabilities.

3.2 Characteristics of the grid

The grid contains 100 elements of the type SEG3.

3.3 Sizes tested and results

One tests displacement in the corner high left of the plate.

Identification	Type of reference	Value of reference	Precision
Not A - DX	'ANALYTICAL'	63.9488	0.1%
Not B - DX	'ANALYTICAL'	32,000	0.1%
Not C - DX	'ANALYTICAL'	0.05120	0.05
Not A - DRZ	'ANALYTICAL'	0.06583	0.05
Not B - DRZ	'ANALYTICAL'	41,133	0.1%
Not B - NYY	'ANALYTICAL'	2.0000	0.05
Not $B1$ - NYY	'ANALYTICAL'	3.84429	0.1%
Not $B1$ - MXX	'ANALYTICAL'	$-4.01497\ 10^{-2}$	0.1%

4 Summary of the results

The various curves show that displacements as well as the calculated efforts are very close to the analytical solution. This test thus makes it possible well to validate the axisymmetric formulation of hull COQUE_AXIS.