SSLS503 - Plate laminated in antisymmetric inflection stacking simply supported

Summary:

This test represents a quasi-static calculation of a laminated plate, in antisymmetric inflection stacking, simply supported, subjected to a pressure uniformly distributed.

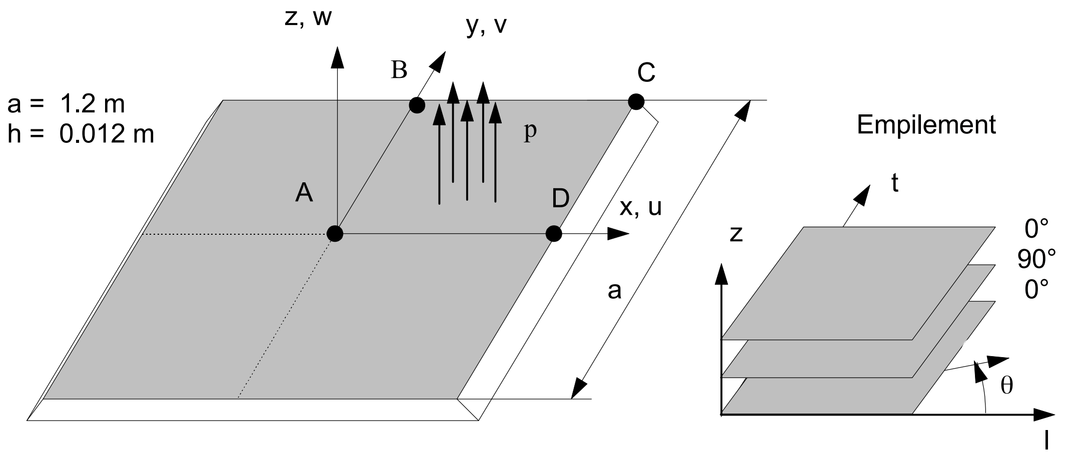
4 modelings make it possible to validate:

- modelings finite elements `DKT` (`QUAD4`, `TRIA3`) and `DST` (`QUAD4`, `TRIA3`) in the case of a composite material (3 different layers of orientation),
- stresses shear transverse.

Displacements and the constraints obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of material

The properties of material constituting the plate are the following ones:

One-way (U) :

$E_l = 4.10^{10}\,Pa$ $\quad E_t = 0.16\,10^{10}\,Pa$ $\quad (\ l \Leftrightarrow x\ ;\ t \Leftrightarrow y\ )$

$G_{lt} = G_{lz} = 8.10^8\,Pa$ $\quad G_{tz} = 3.2\,10^8\,Pa$

$\nu_{lt} = 0.25$

Stacking:

- Orientation: $[0/90/0]$
- Nature: $[U/U/U]$
- Thickness: $[h/3/h/3/h/3]$

1.3 Boundary conditions and loadings

- The plate is simply supported on its contour
- Pressure uniformly distributed : $p = 3000\,Pa$

1.4 Initial conditions

Without object

2 Reference solution

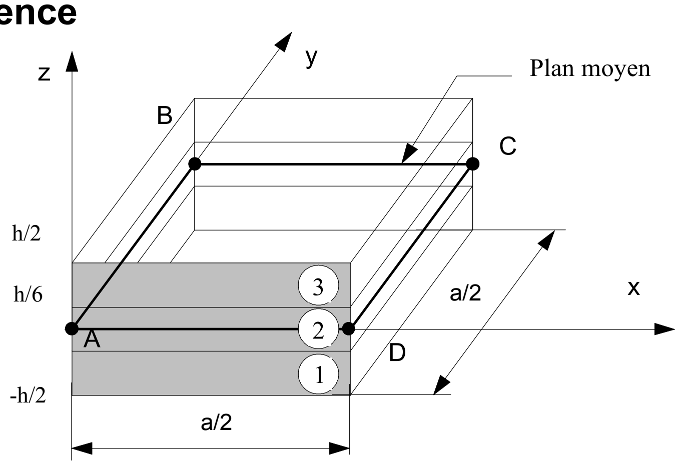
2.1 Method of calculating used for the reference solution

Displacement: analytical solution obtained by decomposition in series of the form:

$$w=\sum_i \sum_j w_{ij} \sin\left(\frac{i\pi x}{a}\right)\sin\left(\frac{j\pi y}{b}\right)$$

Constraints: digital solution [bib1], [bib2]

2.2 Results of reference



The results of reference are the following:

$w(0,0,0)$	$0.01507\,m$	Displacement w in the center of the plate (not A),
$SIXX(0,0,h/2)$	$2.4216\,10^7\,Pa$	Constraint σ_{xx} on the higher skin of the layer 3 ($z=h/2$) in the center of the plate (not A),
$SIYY(0,0,h/6)$ sleep with $90°$	$5.7810\,10^6\,Pa$	Constraint σ_{yy} on the higher skin of the layer 2 ($z=h/6$) in the center of the plate (not A),
$SIXY(a/2,a/2,h/2)$	$1.2825\,10^6\,Pa$	Constraint σ_{xy} at the point C on the higher skin of the layer 3 ,
$SIXZ(a/2,0,0)$	$2.3526\,10^5\,Pa$	Constraint σ_{xz} at the point D on the average skin of the layer 2 ($z=0$),
$SIYZ(0,a/2,0)$	$8.8950\,10^4\,Pa$	Constraint σ_{yz} at the point B on the average skin of the layer 2 ($z=0$),

2.3 Uncertainties on the solution

- The reference solution is given for a number of terms in the series equal to 25.
- The factor of correction of transverse shearing used is of 5/6.
- With an important twinge ($a/h=100$) , the transverse level of shearing is weak and thus difficult to obtain with precision. There exists an uncertainty then on the values of constraint σ_{ij} calculated during the validation of the test $VPCS$, the differences obtained by the software on the components of shearing are about 10% .

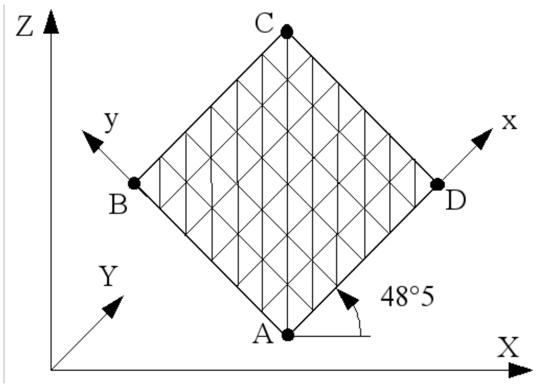
2.4 Bibliographical references

1) VPCS: Software package of composite structural analysis; Examples of validation. Review of the composites and advanced materials, Volume 5 - number except series 1995. Hermes edition.

2) PUTCHA, N.S. and REDDY, J.N. : With mixed shear flexible finite element for the analysis of laminated punts, computer meth. in applied mech. Eng. 44 (1984).

3 Modeling A

3.1 Characteristics of modeling



Modeling DKT (TRIA3)

- The plate is located in the plan $Y=0.5$
- Not A $(0.4\,;0.5\,;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local reference mark)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

3.2 Characteristics of the grid

Many nodes: 49
Many meshs and types : 72 TRIA3

3.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance $(\%)$
$w(0,0,0)$	ANALYTICAL	0.01507	1.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 10^7	2.1
$SIYY(0,0,h/6)$ sleep with $90°$	SOURCE_EXTERNE	5.7810 10^6	2.7
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10^6	4.6
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	–2.3526 10^5	37
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 10^4	3.1

3.4 Remarks

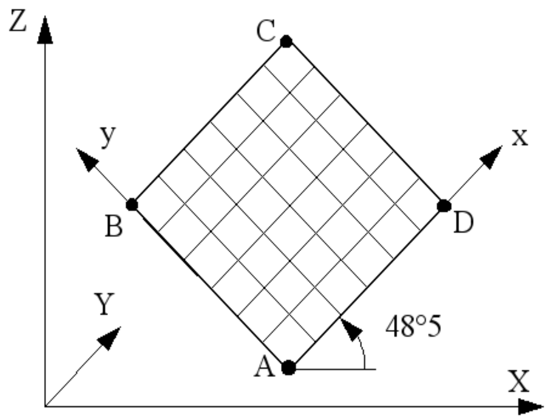
The constraints are expressed in the reference mark of orthotropism defined by ANGL_REP (AFFE_CARA_ELEM), and by the normal of the element.

Components $SIXX$, $SIYY$ and $SIYZ$ are the median values of the two convergent meshs at the points A and C .

The variation obtained on $SIXZ$ is due unlike modeling of transverse shearing: in the reference, one uses a coefficient of transverse correction of shearing of 5/6. In *Code_Aster*, one calculates the distribution of shearings in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

4 Modeling B

4.1 Characteristics of modeling



Modeling `DKT` (`QUAD4`)

- The plate is located in the plan $Y=0.5$
- Not A $(0.4\,;0.5\,;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local reference mark)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

4.2 Characteristics of the grid

Many nodes: 49
Many meshs and types: 36 `QUAD4`

4.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance (%)
$w(0,0,0)$	`ANALYTICAL`	0.01507	1.1
$SIXX(0,0,h/2)$	`SOURCE_EXTERNE`	2.4216 10^7	1.1
$SIYY(0,0,h/6)$ sleep with $90°$	`SOURCE_EXTERNE`	5.7810 10^6	1.1
$SIXY(a/2,a/2,h/2)$	`SOURCE_EXTERNE`	1.2825 10^6	5.1
$SIXZ(a/2,0,0)$	`SOURCE_EXTERNE`	–2.3526 10^5	16
$SIYZ(0,a/2,0)$	`SOURCE_EXTERNE`	8.8950 10^4	4.1

4.4 Remarks

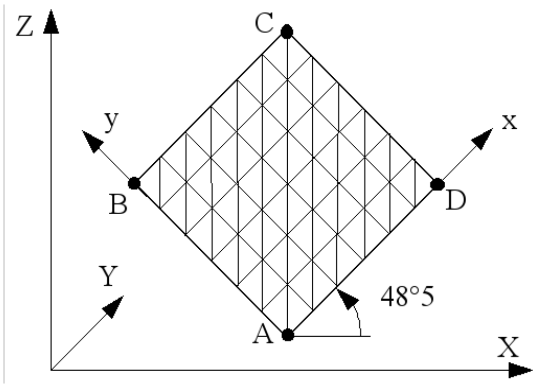
Components $SIXX$, $SIYY$ and $SIYZ$ are the median values of the two convergent meshs at the points A and C .

The variation obtained on $SIXZ$ is due unlike modeling of transverse shearing: in the reference, one uses a coefficient of transverse correction of shearing of 5/6. In *Code_Aster*, one calculates the distribution of shearings in the thickness, presumedly parabolic in each layer.

The sign of $SIXZ$ is opposed to that of the reference solution.

5 Modeling C

5.1 Characteristics of modeling



Modeling DST (TRIA3)

- The plate is located in the plan $Y=0.5$
- Not A $(0.4;0.5;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local reference mark)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

5.2 Characteristics of the grid

Many nodes: 49
Many meshs and types: 72 TRIA3

5.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance $(\%)$
$w(0,0,0)$	ANALYTICAL	0.01507	2.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 10^7	7.1
$SIYY(0,0,h/6)$ sleep with $90°$	SOURCE_EXTERNE	5.7810 10^6	24.
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10^6	4.1
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	–2.3526 10^5	37
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 10^4	26

5.4 Remarks

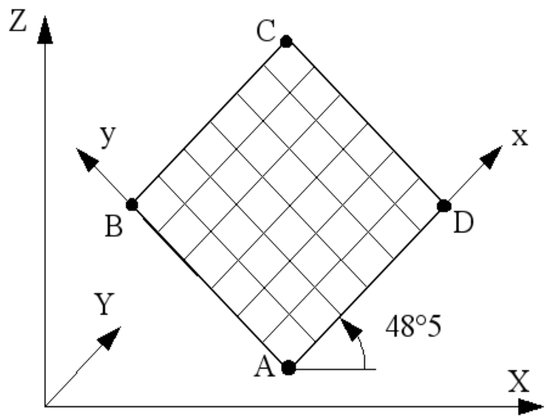
Components $SIXX$, $SIYY$ and $SIYZ$ are the median values of the two convergent meshs at the points A and C .

The variation obtained on $SIXZ$ is due unlike modeling of transverse shearing: in the reference, one uses a coefficient of transverse correction of shearing of 5/6. In *Code_Aster*, one calculates the distribution of shearings in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

The other variations are probably due to the anisotropy of the triangular grid.

6 Modeling D

6.1 Characteristics of modeling



Modeling DST (QUAD4)

- The plate is located in the plan $Y=0.5$
- Not A $(0.4\,;0.5\,;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local reference mark)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

6.2 Characteristics of the grid

Many nodes: 49
Many meshs and types: 36 QUAD4

6.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance $(\%)$
$w(0,0,0)$	ANALYTICAL	0.01507	1.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 10^7	1.1
$SIYY(0,0,h/6)$ sleep with $90°$	SOURCE_EXTERNE	5.7810 10^6	1.1
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10^6	7.1
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	–2.3526 10^5	15.
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 10^4	2.1

6.4 Remarks

Components $SIXX$, $SIYY$ and $SIYZ$ are the median values of the two convergent meshs at the points A and C .

The variation obtained on $SIXZ$ is due unlike modeling of transverse shearing: in the reference, one uses a coefficient of transverse correction of shearing of 5/6. In *Code_Aster*, one calculates the distribution of shearings in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

7 Summary of the results

- **Dieplacings** : some is modeling used (DKT or DST) the results are satisfactory, the maximum error is lower than 0.7% .
- **Plane constraints** : the results are more precise with modeling DKT, the error is lower than 1% except for $SIXY$ (QUAD4) where the error is of 5% . For modeling DST the error is higher ($<8\%$) with an important variation on $SIXX$ (28%) for the mesh TRIA3.
- **Transverse shearing** : some is modeling used (DKT or DST) the results got with the quadrangular grids are closer to the reference solution than those obtained with triangular grids. In the first case the error on the component $SIXZ$ is lower than 15% , and the error on $SIYZ$ is lower than 3% , while in the second case, the error on $SIXZ$ is of 35% and that on $SIYZ$ is understood enters 2% and 24% . Except the least good precision of the triangular grids because of their anisotropy, the variation which remains with quadrangular grids is due unlike modeling of transverse shearing: in the reference, one uses a coefficient of transverse correction of shearing of 5/6. In Code_Aster, one calculates the distribution of shearings in the thickness, presumedly parabolic in each layer.