SSLV109 - Full cylinder in nonuniform pressure mode 1

Summary:

This test validates all the elements of Fourier (triangles and quadrangles of degrees 1 and 2) in elasticity. The features are the following ones:

- variable pressure spaces some,
- imposed displacements,
- matrices of rigidity Fourier mode 1,
- constraints with the nodes Fourier mode 1,
- recombination of Fourier on displacements and constraints (modeling A),
- transverse isotropic material (modeling F).

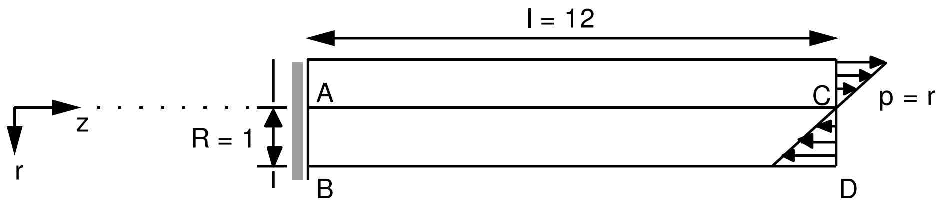
The test has a quadratic analytical solution in displacements.

The interest of the test lies in:

- the comparison between solution calculated and analytical solution on the various finite elements,
- the comparison between the results and Code PERMAS on elements TRIA6 (modeling A).

1 Problem of reference

1.1 Geometry



The modelled field is $ACDB$ (plan $\theta=0$).

1.2 Material properties

$E=72\,N/m^2$

$\nu=0.3$

1.3 Boundary conditions and loadings

$$u_r(A)=u_z(A)=u_\theta(A)=0$$
$$u_z(AB)=0$$

$$p=\bar{p}\frac{r}{R}\cos\theta$$

with $\bar{p}=1.$ and $R=1$ applied in $z=12$.

1.4 Initial conditions

Without object for the static analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

$u_r(r,z,\theta)=u(r,z)\cos\theta$ with $u(r,z)=\frac{M}{2EI}z^2+\frac{\nu\bar{p}}{2ER}r^2$

$u_z(r,z,\theta)=v(r,z)\cos\theta$ with $v(r,z)=-\frac{\bar{p}}{2EI}rz$

$u_\theta(r,z,\theta)=w(r,z)(-\sin\theta)$ with $w(r,z)=\frac{M}{2EI}z^2-\frac{\nu\bar{p}}{2ER}r^2$

All the constraints are worthless except $\sigma_{zz}(r,z)=-\frac{\bar{p}}{R}r$.

The data were selected in such way that $u(x)=u(0,l)=1$.

Displacements are thus written here:

$$u(r,z)=\frac{z^2}{144}+\frac{r^2}{480}\ ;\ v(r,z)=-\frac{rz}{72}\ ;\ w(r,z)=\frac{z^2}{144}-\frac{r^2}{480}$$

and:

$$\sigma_{zz}(r,z)=-r$$

2.2 Results of reference

u, v, w, σ_{zz} in $r=0., 0.5, 1.$; $z=0., 6., 12.$

u_r, u_z, u_θ in $r=0.$; $z=6.$; $\theta=45°$

2.3 Uncertainty on the solution

Analytical solution.

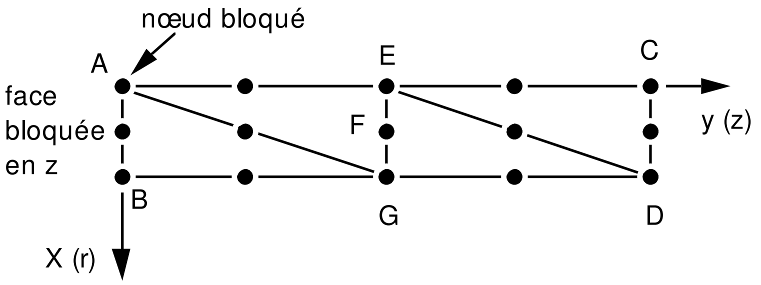
2.4 Bibliographical references

1) PERMAS-HS. Axisymmetric Continued with arbitrary loads. Stuttgart 1985. INTES publication n°224 pp 42 - 49.

3 Modeling A

3.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N3$ $C=N13$ $D=N15$ $E=N7$ $F=N8$ $G=N9$



Limiting conditions:

DDL_IMPO:	(NODE : With	DX = 0. DY = 0. DZ = 0.)
face AB	(GROUP_NO : AB	DY = 0.)

Pressure on the face CD : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(X)=-X$

3.2 Characteristics of the grid

Many nodes: 15

Many meshs and types: 4 TRIA6, 1SEG3 on segment CD

4 Results of modeling A

4.1 Values tested

Node	Size	Reference
B	u	$2.0833\ 10^{-3}$
	v	0.
	w	$-2.0833\ 10^{-3}$
	σ_{zz}	– 1.
E	u	0.25
	v	0.
	w	0.25
	σ_{zz}	0.
F	u	0.250521
	v	– 0.04166
	w	0.0249479
	σ_{zz}	– 0.5
G	u	0.252083
	v	– 0.083333
	w	0.247917
	σ_{zz}	– 1.
C	u	1.
	v	0.
	w	1.
	σ_{zz}	0.
D	u	1.00208
	v	– 0.16666
	w	0.99791
	σ_{zz}	– 1.

4.2 Remarks

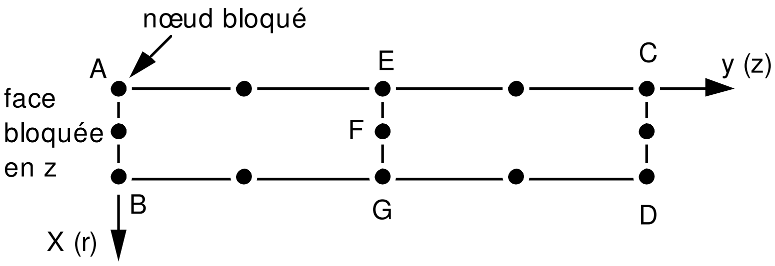
The analytical solution is found with a precision <0.02 for displacements and <0.1 for the constraints.

With a digital formula of integration at 6 points of GAUSS (instead of 3) to calculate the stiffness, one would find the relation with 10^{-10} close (like PERMAS).

5 Modeling B

5.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N3$ $C=N13$ $D=N15$ $E=N7$ $F=N8$ $G=N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With              DX = 0.  DY = 0.  DZ = 0.)
```
face AB (GROUP_NO : AB DY = 0.)

Pressure on the face CD : `PRES_REP (GROUP_MA: End NEAR: p)`

p being defined by `AFFE_CHAR_MECA_F` by $p(X)=-X$

5.2 Characteristics of the grid

Many nodes: 15

Many meshs and types: 2 QUAD8, 1 SEG3 on segment CD

6 Results of modeling B

6.1 Values tested

Node	Size	Reference
B	u	$2.0833\ 10^{-3}$
	v	0.
	w	$-2.0833\ 10^{-3}$
	σ_{zz}	– 1.
E	u	0.25
	v	0.
	w	0.25
	σ_{zz}	0.
F	u	0.250521
	v	– 0.04166
	w	0.0249479
	σ_{zz}	– 0.5
G	u	0.252083
	v	– 0.08333
	w	0.247917
	σ_{zz}	– 1.
C	u	1.
	v	0.
	w	1.
	σ_{zz}	0.
D	u	1.00208
	v	– 0.16666
	w	0.99791
	σ_{zz}	– 1.

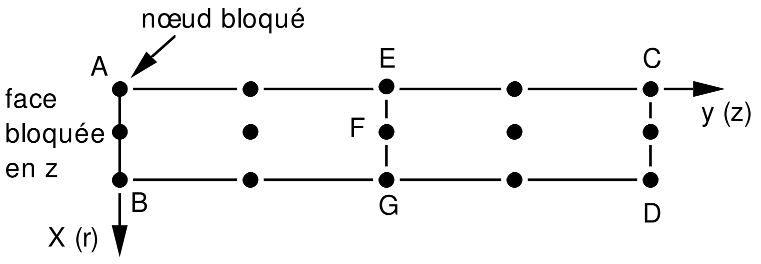
6.2 Remarks

The analytical solution is found with 10 or 11 significant figures.

7 Modeling C

7.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N3$ $C=N13$
$D=N15$ $E=N7$ $F=N8$ $G=N9$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
```
face AB (GROUP_NO : AB DY = 0.)

Pressure on the face CD : PRES_REP (GROUP_MA = End, CLOSE = p)

p being defined by AFFE_CHAR_MECA_F by $p(x)=-x$

7.2 Characteristics of the grid

Many nodes: 15

Many meshs and types: 2 QUAD9, 1 SEG3 on segment CD

8 Results of modeling C

8.1 Values tested

Node	Size	Reference
B	u	$2.0833\ 10^{-3}$
	v	0.
	w	$-2.0833\ 10^{-3}$
	σ_{zz}	– 1.
E	u	0.25
	v	0.
	w	0.25
	σ_{zz}	0.
F	u	0.250521
	v	– 0.04166
	w	0.0249479
	σ_{zz}	– 0.5
G	u	0.252083
	v	– 0.08333
	w	0.247917
	σ_{zz}	– 1.
C	u	1.
	v	0.
	w	1.
	σ_{zz}	0.
D	u	1.00208
	v	– 0.16666
	w	0.99791
	σ_{zz}	– 1.

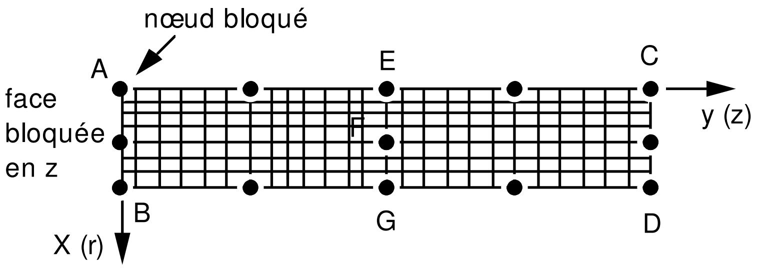
8.2 Remarks

The analytical solution is found with 10 or 11 significant figures.

9 Modeling D

9.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N1129$ $C=N1369$ $D=N2169$ $E=N141$ $F=N705$ $G=N1269$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With              DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB                     DY = 0.)
```

Pressure on the face CD : `PRES_REP (GROUP_MA: End NEAR: p)`

p being defined by `AFFE_CHAR_MECA_F` by $p(x)=-x$

9.2 Characteristics of the grid

Many nodes: 2169

Many meshs and types: 1920 QUAD4, 8 SEG2 on segment CD

10 Results of modeling D

10.1 Values tested

Node	Size	Reference
B	u	$2.0833\ 10^{-3}$
	v	0.
	w	$-2.0833\ 10^{-3}$
	σ_{zz}	– 1.
E	u	0.25
	v	0.
	w	0.25
	σ_{zz}	0.
F	u	0.250521
	v	– 0.04166
	w	0.0249479
	σ_{zz}	– 0.5
G	u	0.252083
	v	– 0.083333
	w	0.247917
	σ_{zz}	– 1.
C	u	1.
	v	0.
	w	1.
	σ_{zz}	0.
D	u	1.00208
	v	– 0.16666
	w	0.99791
	σ_{zz}	– 1.

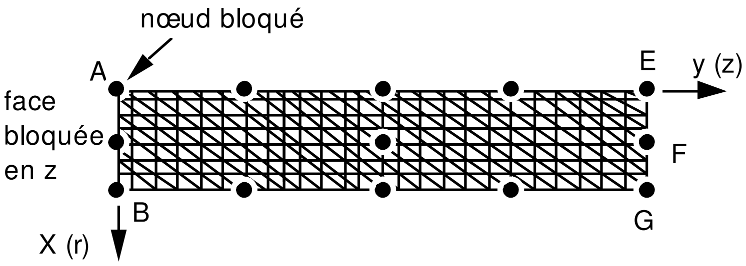
10.2 Remarks

To obtain one precision of about 1% on the constraints, it is necessary to model the structure very finely (8 elements radially and 240 axially).

11 Modeling E

11.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N2421$
$E=N121$ $F=N1331$ $G=N2541$



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
face AB        ( GROUP_NO : AB                 DY = 0.)
```

Pressure on the face EG : PRES_REP (GROUP_MA: End NEAR: p)

p being defined by AFFE_CHAR_MECA_F by $p(x)=-x$

11.2 Characteristics of the grid

Many nodes: 2541

Many meshs and types: 4800 TRIA3, 20 SEG2 on segment EG

11.3 Remarks

To decrease the number of nodes, one modelled the structure for $y \leq 6$.
The precision on the results is nevertheless less than for elements QUAD4.

12 Results of modeling E

12.1 Values tested

Node	Size	Reference
B	u	$2.0833\ 10^{-3}$
	v	0.
	w	$-2.0833\ 10^{-3}$
	σ_{zz}	– 1.
E	u	0.25
	v	0.
	w	0.25
	σ_{zz}	0.
F	u	0.250521
	v	– 0.04166
	w	0.249479
	σ_{zz}	– 0.5
G	u	0.252083
	v	– 0.083333
	w	0.247917
	σ_{zz}	– 1.

12.2 Remarks

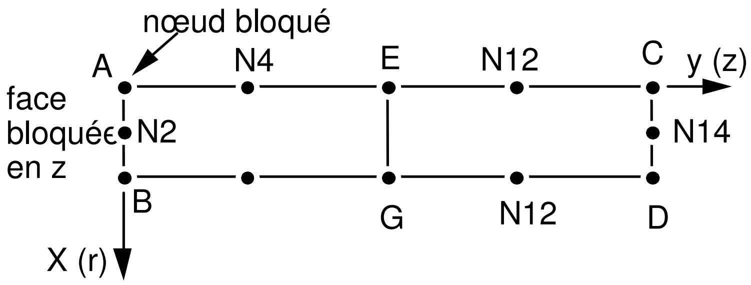
The precision on displacements is lower than 3%, that on the constraints lower than 2%.

On this example, the TRIA3 converge definitely less quickly than the QUAD4 towards the exact solution.

13 Modeling F

13.1 Characteristics of modeling

Number of the nodes: $A=N1$ $B=N3$ C = N13
$D=N15$ $E=N7$ G = N9



Limiting conditions:

```
DDL_IMPO:      ( NODE : With          DX = 0.  DY = 0.  DZ = 0.)
```
face AB `( GROUP_NO : AB            DY = 0.)`

Pressure on the face CD : `PRES_REP (GROUP_MA: End NEAR: p)`

p being defined by `AFFE_CHAR_MECA_F` by $p(X)=-X$

13.2 Characteristics of the grid

Many nodes: 15

Many meshs and types: 2 QUAD8, 1SEG3 on segment CD

14 Results of modeling F

14.1 Values tested

Node	Size	Reference
$N2$	u	2.6041666
	w	– 2.6041666
A	σ_{zz}	0.
B	σ_{zz}	– 1.
$N4$	u	0.0625
	w	0.0625
E	u	0.25
	w	0.25
	σ_{zz}	0.
G	v	– 0.083333
	σ_{zz}	– 1.
$N10$	u	0.5625
	w	0.5625
$N12$	v	– 0,125
C	u	1.
	w	1.
	σ_{zz}	0.
$N14$	v	– 0.083333
D	v	– 0.166666
	σ_{zz}	– 1.

15 Summary of the results

The elements of order 2 give the analytical solution.

The elements of order 1 converge slowly towards the solution and require very fine grids. Times calculations remain however reasonable.