SSLV116 – Circular crack in medium 3D with initial constraints.

Summary:

The purpose of this test is to validate the calculation of the stress intensity factors (SIFs) along the bottom of a crack 3D, within the framework of elasticity in the presence of initial constraints.

This test model a cube embedded on all its faces, presenting a plane central circular crack. It is subjected to an initial stress field due to the application of a thermal field.

This test contains 2 modelings:

- Modeling a: the crack is with a grid (FEM);
- Modeling B (presentE in the file of validation): the crack is not with a grid, it is represented by level-sets (X-FEM).

For two modelings, the stress intensity factors are evaluated by the orders `POST_K1_K2_K3` and `CALC_G.`

The digital values are compared with the values obtained in the case of the thermal loading are equivalent.

1 Problem of reference

1.1 Geometry

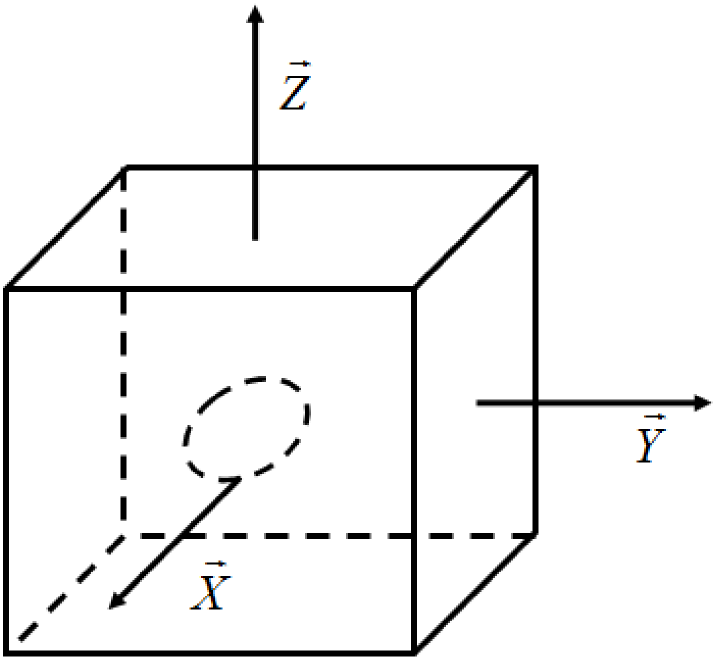

Figure 1.1 - Geometry of the case test.

The crack is to circulairE (penny shaped ace) of ray $a=2\mathrm{m}$ in the plan OXY. The with dimensions one of the cube is length $L=10\mathrm{a}$. Thus, it is considered that the crack is in an infinite medium.

1.2 Material properties

The material, isotropic rubber band, have the properties:

$E=200000\,\mathrm{MPa}$

$\nu=0{,}3$

$\alpha=0{,}00001\,K^{-1}$

1.3 Boundary conditions and loadings

The outsides of volume are embedded.

Taking into account symmetries, one models only the moitié of the structure, namely it half space such as $Y>0$. Conditions of symmetry are thus applied to the face $Y=0$: on this face, following displacement Y is blocked.

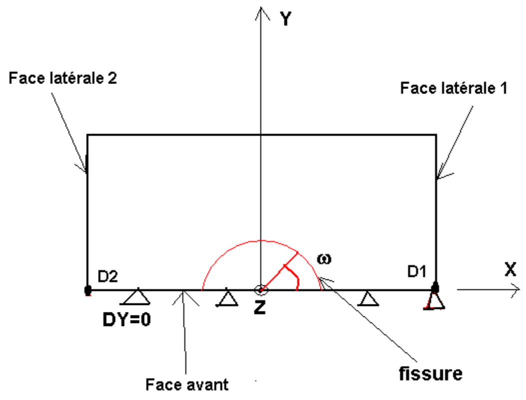

Figure 1.2: Condition of symmetry

The structure, initially of uniform temperature 250°C, is subjected to a field of variable temperature in space, according to Z. the structure cools, but is embedded of all shareS. So constraints of opening are created on the crack.

The Figure below presents the profile of temperature imposed.

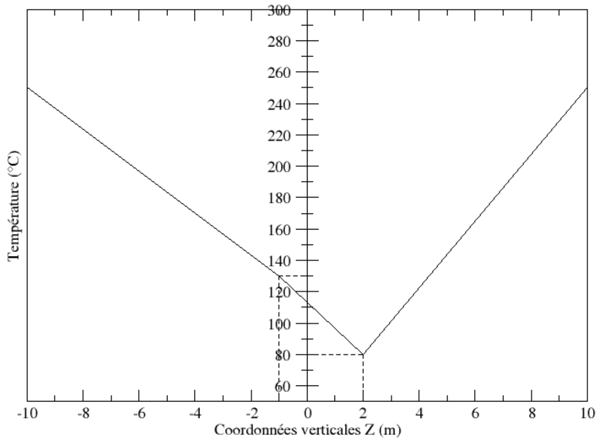

Figure 1.3 - Field of temperature imposed.

L'application of this field of temperature generate constraints (closed crack). These constraints are then extracted and constituteNT an initial stress field for the calculation of the factors of intensity of the constraints.

2 Reference solution

The reference solution is determined by calculation of the factors of intensity of the constraints directly starting from the thermal loading.

The validation is made starting from the maximum of the rate of refund of energy room (in each node of the face of crack) and of the factor of intensity of the constraints room to 90° of the crack.

3 Modeling A

3.1 Characteristics of modeling

In this modeling, the crack is with a grid (case FEM). The grid comprises a torus surrounding the bottom of crack.

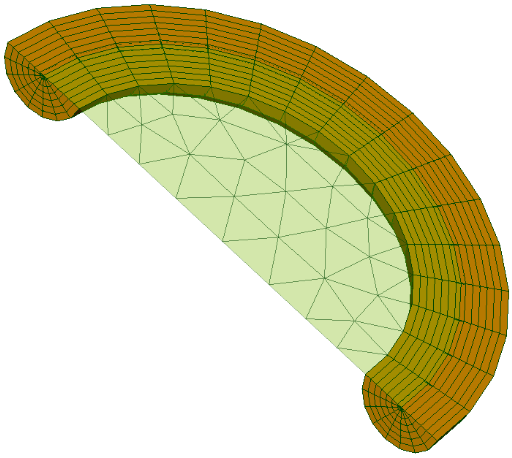

Figure 3.1: radiant grid

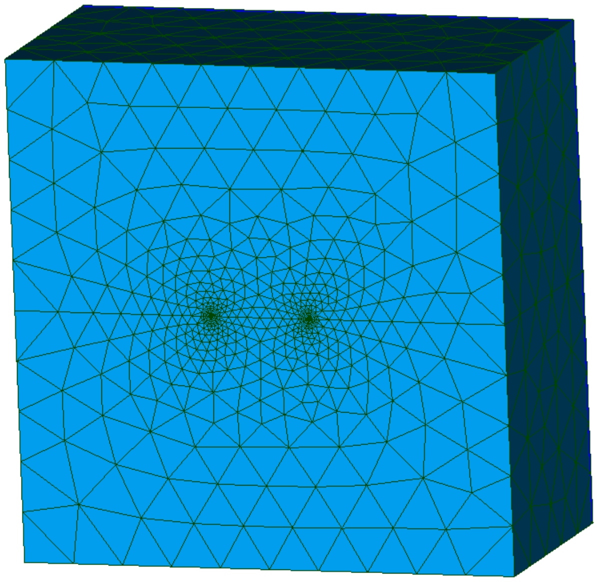

Figure 3.2: grid of the structure

3.2 Characteristics of the grid

Many nodes: 26673

Number of meshs and type: 16 SEG3, 1540 TRIA6, 416 QUAD8, 12818 TETRA10, 256 PENTA15, 256 PYRAM13, and 1536 HEXA20.

On the level of the torus (figure 3.1), the length of the prisms in the radial direction is of $h_{pris}=0{,}04\,m$ (layer of PENTA15 connected to the bottom of crack), and the length of the hexahedrons in the radial direction is of $h_{hexa}=0{,}06\,m$ (6 layers D'elements HEXA20).

The nodes mediums of the edges of meshs concerning the bottom of crack (PENTA15) are moved with the quarter of these edges (elements of Barsoum).

3.3 Calculations carried out

It is pointed out that LE principle of this test of checking equivalence enters the taking into account of an initial state and a thermal loading are equivalent, during a postprocessing of breaking process.

The sizes of interest are the rate of refund of energy (G) and factors of intensity of the constraints (limited here to K_I because the configuration requests only mode I because of symmetry). The paragraph §Error: Reference source not found detail the various manners post-of treating these sizes which are used and tested.

Modeling : The configuration is bidimentionnelle. Mechanical calculations are carried out under assumption of Plane Deformations.

Calculation of reference:
The analytical solution not being known, the first stage consists in generating calculation serving as reference: i.e. calculation in linear thermoelasticity. One connect creation of the field of temperature

then LE calculation thermO-rubber band liénaire, after which two types of postprocessings are carried out:
- a postprocessing in breaking process in order to determine the values of reference of the sizes of interest,
- an extraction of the stress field, which will be used as initial stress field.

Calculation with initial constraints:
A calculation in incremental linear elasticity with an initial state given in the shape of an initial stress field, with presence of the crack, is carried out. It is important to stress that **the initial state is well informed *via* an initial stress field**, and not by the data of a concept of type result (EVOL_NOLI) in the operator of resolution.
Then, postprocessings in breaking process are carried out. In order to test the various opportunities given by the order CALC_G, several configurations are put in work, which differ by the type of initial stress field provided to the order CALC_G : at the points of Gauss, with the nodes by element or the nodes.

Three crowns of integration of the field theta for the order CALC_G are used:

- Crown 1 (2h-4h) : $\mathrm{RINF\_1} = h_{pris} + h_{hexa}$ and $\mathrm{RSUP\_1} = h_{pris} + 3\,h_{hexa}$;
- Crown 2 (4h-6h) : $\mathrm{RINF\_2} = h_{pris} + 3\,h_{hexa}$ and $\mathrm{RSUP\_2} = h_{pris} + 6\,h_{hexa}$;
- Crown 3 (2h-6h) : $\mathrm{RINF\_3} = h_{pris} + h_{hexa}$ and $\mathrm{RSUP\_3} = h_{pris} + 6\,h_{hexa}$.

A smoothing of the type is chosen LEGENDRE.

The parameter ABS_CURV_MAXI of the operator POST_K1_K2_K3 is selected so as to retain 5 nodes on the segment of extrapolation.

3.4 Sizes tested

One tests Lbe valueS rate of refund of energy (G) resulting from:
- the operator CALC_G , option CALC_G ,
- the operator CALC_G, option CALC_K_G.

One also tests the values of the factor of intensity of the constraints in mode I (K_I) exits of:
- the operator CALC_G, option CALC_K_G,
- the operator POST_K1_K2_K3.

For postprocessings with the operator CALC_G, 3 crowns previously defined are systematically tested.

Lastly, one compared with each time, values resulting from the calculation of reference to the values resulting from calculation with initial state, to Iso-condition (even representation of the initial stress field, even operator of postprocessing, even crown).

For G and K_I resulting from the operator CALC_G, the max of G is tested K_I along the face of the crack.

K_I of POST_K1_K2_K3 is tested only at the point such as $\omega = 90°$ (see figure 1.2).

Test of the rates of refund of energy:

Identification	Reference	Type of reference	Tolerance
CALC_G			
$max(G)$ resulting from thermal calculation (reference), crown n°1	-	'NON_REGRESSION'	-
$max(G)$ resulting from thermal calculation (reference), crown n°2	1923020	'AUTRE_ASTER'	0.03%
$max(G)$ resulting from thermal calculation (reference), crown n°2	1923020	'AUTRE_ASTER'	0.04%
$max(G)$ with definite initial state at the points of Gauss, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with definite initial state at the points of Gauss, crown n°2	1922612	'AUTRE_ASTER'	0, 04%
$max(G)$ with definite initial state at the points of Gauss, crown n°3	1922816	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes by element, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with initial state defined in the nodes by element, crown n°2	1922612	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes by element, crown n°3	1922816	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with initial state defined in the nodes, crown n°2	1922612	'AUTRE_ASTER'	0, 06%
$max(G)$ with initial state defined in the nodes, crown n°3	1922816	'AUTRE_ASTER'	0, 05%

Identification	Reference	Type of reference	Tolerance
CALC_K_G			
$max(G)$ resulting from thermal calculation (reference), crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ resulting from thermal calculation (reference), crown n°2	1922612	'AUTRE_ASTER'	0, 03%
$max(G)$ resulting from thermal calculation (reference), crown n°3	1922816	'AUTRE_ASTER'	0, 04%
$max(G)$ with definite initial state at the points of Gauss, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with definite initial state at the points of Gauss, crown n°2	1922612	'AUTRE_ASTER'	0, 05%
$max(G)$ with definite initial state at the points of Gauss, crown n°3	1922816	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes by element, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with initial state defined in	1922612	'AUTRE_ASTER'	0, 05%

the nodes by element, crown n°2			
$max(G)$ with initial state defined in the nodes by element , crown n°3	1922816	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes, crown n°1	1923020	'AUTRE_ASTER'	0,02%
$max(G)$ with initial state defined in the nodes, crown n°2	1922612	'AUTRE_ASTER'	0, 04%
$max(G)$ with initial state defined in the nodes, crown n°3	1922816	'AUTRE_ASTER'	0, 04%

Test of the stress intensity factors:

Identification	Reference	Type of reference	Tolerance
CALC_K_G			
$K_I(90°)$ resulting from thermal calculation (reference), crown n°1	-	'NON_REGRESSION'	-
$K_I(90°)$ resulting from thermal calculation (reference), crown n°2	663418796	'AUTRE_ASTER'	2 %
$K_I(90°)$ resulting thermal calculation (reference), crown n°3	663418796	'AUTRE_ASTER'	1 %
$K_I(90°)$ with definite initial state at the points of Gauss, crown n°1	663418796	'AUTRE_ASTER'	0,01
$K_I(90°)$ with definite initial state at the points of Gauss, crown n°2	673882118	'AUTRE_ASTER'	0,01
$K_I(90°)$ with definite initial state at the points of Gauss, crown n°3	668650457	'AUTRE_ASTER'	0,01
$K_I(90°)$ with initial state defined in the nodes by element, crown n°1	663418796	'AUTRE_ASTER'	0,01
$K_I(90°)$ with initial state defined in the nodes by element, crown n°2	673882118	'AUTRE_ASTER'	0,01
$K_I(90°)$ with initial state defined in the nodes by element, crown n°3	668650457	'AUTRE_ASTER'	0,01
$K_I(90°)$ with initial state defined in the nodes , crown n°1	663418796	'AUTRE_ASTER'	0,011
$K_I(90°)$ with initial state defined in the nodes, crown n°2	673882118	'AUTRE_ASTER'	0,01
$K_I(90°)$ with initial state defined in the nodes, crown n°3	668650457	'AUTRE_ASTER'	0,01

Identification	Reference	Type of reference	Tolerance
POST_K1_K2_K3			
$K_I(90°)$ resulting from thermal calculation (reference)	-	'NON_REGRESSION'	-

$K_I(90°)$ with initial state	621498251	'AUTRE_ASTER'	0.01%

4 Modeling B

4.1 Characteristics of modeling

In this modeling, the crack is not with a grid (case X-FEM).

One uses same Mayllage that for the case test sslv154b.

In order to obtain the bestE precision on the results, the free initial grid was refined on the level of the bottom of crack using the order MACR_ADAP_MAIL.

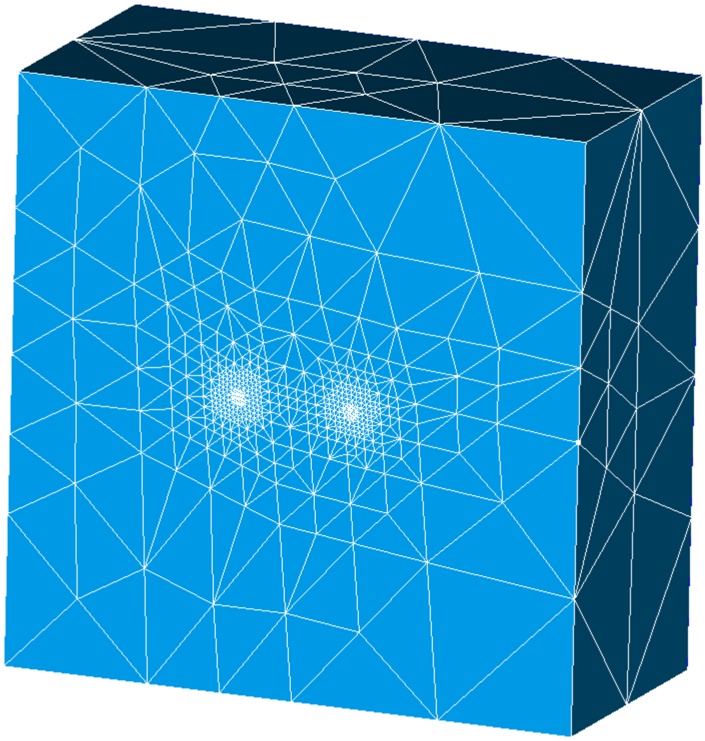

Figure 4.1: refined grid of the structure

4.2 Characteristics of the grid

Many nodes: 1146

Number of meshs and type: 64573 TETRA4

The length characteristic of an element close to the bottom to crack is of $0{,}07\,m$.

4.3 Results

Three crowns of integration of the field theta for the order CALC_G are used:

- Crown 1: $\mathrm{RINF}=0{,}12\,m$ and $\mathrm{RSUP}=0{,}528\,m$.
- Crown 2: $\mathrm{RINF\_2}=0{,}5\,\mathrm{RINF}$ and $\mathrm{RSUP\_2}=0{,}5\,\mathrm{RSUP}$
- Crown 3: $\mathrm{RINF\_2}=1{,}5\,\mathrm{RINF}$ and $\mathrm{RSUP\_2}=1{,}5\,\mathrm{RSUP}$

A smoothing of the type is chosen LEGENDRE.

The parameter ABS_CURV_MAXI of the operator POST_K1_K2_K3 is selected so as to retain 5 nodes on the segment of extrapolation.

K_I is tested only at the point such as $\omega=90°$.

Option of calculation	Case considered	Crown	Max G (J/m ²)	K1 (90°) $\left(MPa.m^{\frac{1}{2}}\right)$
POST_K1_K2_K3	Thermics			639, E6
	Initial constraint			612E6
CALC_G	Thermics		2,06E6	
	Initial constraint	1	2,03E6	
		2	1,95E6	
		3	2,06E6	
CALC_K_G	Initial constraint	1	1,92E6	677E6
		2	1,94E6	661E6
		3	2,07E6	688E6

Table 1- Results of modeling FEM.

- It is noted whereas the results are:
- In conformity with the references to 6% for G and 8% for K1. Let us note that the reference POST_K1_K2_K3 present a variation of 4% between the thermal case and the case with initial constraints
- Different from 5% maximum for G enters the options CALC_G and CALC_K_G,
- Independent of the crown with 5% near for G of CALC_G, 7% for G of CALC_K_G and 4% for K1.

4.4 Sizes tested

As for modeling A, one tests the value maximale rates of refund of energy and the factor of intensity of the constraints for an angle of 90°, 3 crowns, in the calculation cases thermal and initial state.

However, in X-FEM, the constraints can be given only to the nodes, which limits the number of tests.

One tests many sizes, in order to control the thermal reference and the initial state.

Identification	Option of calculation	Notice	Type of reference	Value of reference	% Tolerance
$max(Gref)$	CALC_G, thermics	couronne1	'NOT REGRESSION'	$2.06919\ 10^{6}$	1%
$max(Gini)$	CALC_G, initial constraint	Couronne1	'ANOTHER ASTER'	$2.06919\ 10^{6}$	2%
$max(Gini2)$	CALC_G, initial constraint	Couronne2	'ANOTHER ASTER'	$2.06919\ 10^{6}$	6%
$max(Gini3)$	CALC_G, initial constraint	Couronne3	'ANOTHER ASTER'	$2.06919\ 10^{6}$	2%
$max(Girini)$	CALC_K_G, initial constraint	Couronne1	'ANOTHER ASTER'	$2.06919\ 10^{6}$	7 %
$max(Girini2)$	CALC_K_G, initial constraint	Couronne2	'ANOTHER ASTER'	$2.06919\ 10^{6}$	7 %
$max(Girini3)$	CALC_K_G, initial constraint	Couronne3	'ANOTHER ASTER'	$2.06919\ 10^{6}$	3 %
$max(Girini4)$	CALC_K_G, initial constraint	Couronne1 initial constraint Node	'ANOTHER ASTER'	$1.92013\ 10^{6}$	1.5 %
$max(Girini5)$	CALC_K_G, initial constraint	Couronne2 initial constraint Node	'ANOTHER ASTER'	$1.95458\ 10^{6}$	1.5 %
$max(Girini6)$	CALC_K_G, initial constraint	Couronne3 initial constraint Node	'ANOTHER ASTER'	$1.93471\ 10^{6}$	1.5 %
$Kref(90°)$	POST_K1_K2_K3 Thermics		'NOT REGRESSION'	$6.397943\ 10^{8}$	1 %

Girini $K_I(90°)$	CALC_K_G, initial Constraint	Couronne1,	'ANOTHER ASTER'	6.397943 10^8	7%
Girini2 $K_I(90°)$	CALC_K_G, initial Constraint	Couronne2 initial constraint Gauss	'ANOTHER ASTER'	6.397943 10^8	5%
Girini3 $K_I(90°)$	CALC_K_G, initial Constraint	Couronne3 initial constraint Gauss	'ANOTHER ASTER'	6.397943 10^8	8%

5 Summary of the results

This CAS-test validates the calculation of the stress intensity factors of a crack 3D in the presence of initial state.

Lmodeling by one has crack with a grid is more satisfactory than modeling X-FEM. It shows in particular one dependence less with the crown.

More precisely in FEM (modeling A):

- Conclusions on G:
  - Very good independence results of CALC_G with the crowns (maximum difference between crowns lower than 0.04%) , in all the cases,
  - Strictly identical results between an initial stress field provided to CALC_G of type ELGA and ELNO (what is logical because the field of temperature is linear per piece),
  - Close results with the reference, with identical crown (variation D E 0.02 % to 0.0 6 % according to the configurations) ,
  - Strictly has ucune difference enters CALC_G and CALC_K_G (with 6 significant figures)
- Conclusions on K:
  - conclusions are not also clear of in 2d (cf sslp115a)
  - In particular, one observes less good independence results of CALC_K_G with the crowns: maximum difference between crowns inferior between 1 % and 2 % in the case of reference, variation which also remains for calculation with initial state,
  - However, Résultats with Initial State very close relations of the reference, with identical crown (variation of about 0.01 %),