SSLV121 - Stretching of an isotropic parallelepiped transverse under its own weight

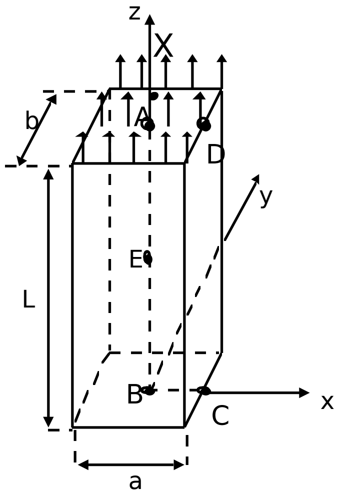
Summary:

This test of mechanics of the structures allows the evaluation of displacements and the constraints of a parallelepiped becoming deformed under its own weight. The material is elastic linear isotropic transverse. Modeling is three-dimensional. The model is similar to test VPCS SSLV07 (but in this case the material is isotropic) and with test SSLV120 (in this case the material is orthotropic.).

Variations of the results got by *Aster* are located between $0{,}00\%$ and $0{,}4\%$ analytically calculated reference.

1 Problem of reference

1.1 Geometry



Hauteur : L = 3 m Largeur : a = 1 m Epaisseur : b = 1 m

Coordinates of the points (in meters):

	A	B	C	D	E	X
x	0.	0.	0.5	0.5	0.	0.
y	0.	0.	0.	0.	0.	0.5
z	3.	0.	0.	3.	1.5	3.

1.2 Material properties

YOUNG moduli in the plan xy and direction z :

$$E_L = 5.10^{11} Pa \text{ , } E_N = 2.10^{11} Pa \text{ .}$$

Poisson's ratios relating to the plan xy and with the direction z :

$$\nu_{\mathrm{LT}} = 0.1 \text{ , } \nu_{\mathrm{LN}} = 0.3 \text{ .}$$

Modulus of rigidity relating to the direction z :

$$G_{\mathrm{LN}} = 7.69231\ 10^{10} Pa \text{ .}$$

Density: $\rho = 7800\, kg/m^3$.

1.3 Boundary conditions and loadings

Not A : ($u = v = w = 0$, $\theta_x = \theta_y = \theta_z = 0$)

Actual weight following the axis z : $\rho g z$
Uniform constraint with traction for the higher face:

$$\sigma_z = \rho\, g\, L = +229\,554.\, Pa$$

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from that given in card SSLV07/89 of guide VPCS (while considering in more one transverse isotropic elastic matrix). The analytical expression of the solution is the following one:

Displacements:

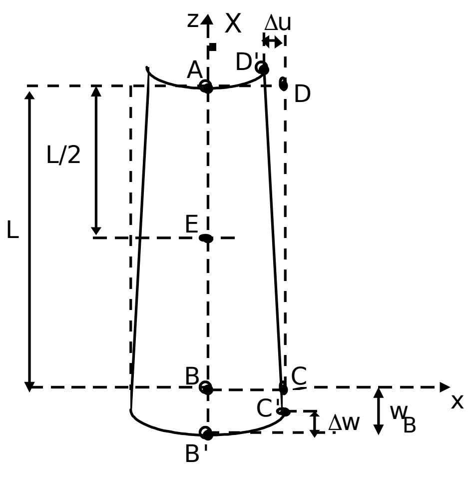
$$u=-\frac{\nu_{NL}\rho g x z}{E_N}$$

$$v=-\frac{\nu_{NL}\rho g y z}{E_N}$$

$$w=\frac{\rho g z^2}{2E_N}+\frac{\rho g \nu_{NL}}{2E_N}(x^2+y^2)-\frac{\rho g L^2}{2E_N}$$

Constraints:

$$\sigma_{zz}=\rho g z \quad \sigma_{zz}=\sigma_{yy}=\sigma_{xy}=\sigma_{yz}=\sigma_{zx}=0$$



2.2 Results of reference

Displacement of the points B , C , D , E and X .

Constraints σ_{zz} in A and E

2.3 Uncertainty on the solution

Exact analytical results.

2.4 Bibliographical references

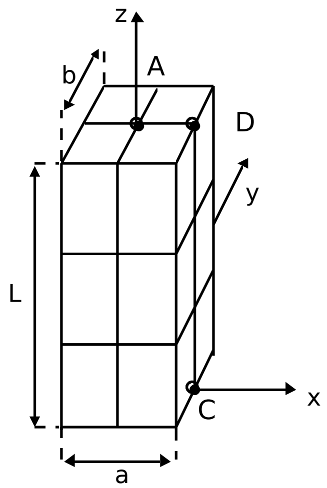
1 TIMOSHENKO (S.P) Theory of elasticity - Paris - Polytechnic Bookstore CH. Béranger, p.279 with 282 (1961)

2 S.W. TSAI, H.T. HAHN - Introduction to composite materials. Technomic Publishing Company (1980).

3 Modeling A

3.1 Characteristics of modeling

`3D`



Cutting:

3 elements in height

2 elements in width and thickness

meshs `hexa20`

Limiting conditions:

on the axis AB	`DDL_IMPO:`	`(GROUP_NO: ABsansA DX=0., DY=0. )`
in A and D		`(NODE: With DX=0., DY=0., DZ=0. )`
		`(NODE: D DY=0.)`

Names of the nodes:	$A=N59$	$B=N53$	$C=N12$
	$D=N18$	$E=N56$	$X=N70$

3.2 Characteristics of the grid

Many nodes: 111

Many meshs and types: 12 `HEXA20`

3.3 Sizes tested and results

Identification	Reference	*Aster*	% difference
U_B	0.	10^{-22}	-
V_B	0.	10^{-22}	-
W_B	-1.72165510^{-6}	$-1.721674\ 10^{-6}$	0,001
U_C	0.	$= 10^{-14}$	-
V_C	0.	$= 10^{-19}$	-

W_C	$-1.715916\ 10^{-6}$	$-1.715935\ 10^{-6}$	0,001
U_D	$-6.88662\ 10^{-8}$	$-6.88653\ 10^{-8}$	0,001
V_D	0.	$= 10^{-23}$	-
W_D	$5.73885\ 10^{-9}$	$5.71514\ 10^{-9}$	0,413
U_E	0.	$= 10^{-22}$	
V_E	0.	$= 10^{-23}$	
W_E	$-1.291241\ 10^{-6}$	$-1.291260\ 10^{-6}$	0,002
(Pa)			
$\sigma_{zz}\ (A)$	$2.29554\ 10^{5}$	$2.2956\ 10^{5}$	< 0.01
$\sigma_{zz}\ (E)$	$1.14777\ 10^{5}$	$1.14777\ 10^{5}$	< 0.01
$\sigma_{zz}\ (X)$	$2.29554\ 10^{5}$	$2.29549\ 10^{5}$	< 0.01
U_X	0.	10^{-20}	-
V_X	$-6.88662\ 10^{-8}$	$-6.886534\ 10^{-8}$	-
W_X	$5.73885\ 10^{-8}$	$5.71514\ 10^{-9}$	0,413

Modeling in `HEXA20` is completely acceptable for this coarse grid.

4 Summary of the results

The results concerning displacements and the constraints are very close to the analytical solution with adopted modeling ($< 0.2\%$ for displacements, $< 0.5\%$ for the constraints).

The fact that there is only one component of the constraints (σ_{zz}) in the problem allows to test only 2 elastic coefficients (E_N and ν_{LN}).

Although these coefficients are constant, they were introduced in the form of functions to test the functionality ELAS_ISTR_FO.

Elastic coefficients in the plan XY and direction Z were selected so as to obtain the same values of displacements at the points B , C , D and E that those calculated for an isotropic material (test SSLV07) or orthotropic (test SSLV120). Numerically, these values are very close to those of these tests at the points considered (about 10^{-6}) the difference resulting from the method of construction of the matrices from stiffness in the various cases. At the point X , these values differ but correspond well to the reference solution.