

SSLV134 - Circular crack in infinite medium

Summary

This test allows, after obtaining the field of displacement by `MECA_STATIQUE`, the calculation of the rate of refund of energy room for a circular crack plunged in a presumedly infinite medium.

For the first modeling, only a half space defined by the plan of the crack is represented. The bottom of crack is then a closed curve (a circle) and is defined as such in `DEFI_FOND_FISS`. The local rate of refund is compared to the analytical solution of reference.

Seven following modelings make it possible to calculate the stress intensity factors K_I and K_3 , in 3D and axisymmetric, calculated by `POST_K1_K2_K3` and/or `CALC_G`.

- Modeling A tests G for a grid 3D with the closed crack,
- Modeling B tests K_I for a grid 3D ,
- Modeling C tests K_I for an axisymmetric grid,
- Modeling D tests the combination of K_I and K_3 for a grid 3D .
- Modeling H tests K_I for a crack nonwith a grid (method X-FEM)
- Modeling I tests K_I for an axisymmetric crack nonwith a grid (method X-FEM)
- Modeling J tests G for a grid 3D with the crack closed for the incompressible elements,
- Modeling K tests K_I for an axisymmetric grid for the incompressible elements,
- Modeling L tests G for a grid 3D with the closed crack (method X-FEM).

1 Problem of reference

1.1 Geometry

The crack is circular (penny shaped ace) of ray a , in the plan Oxy . So that the medium is regarded as infinite, the sizes characteristic of the solid mass are about 5 times higher than the ray a .

1.2 Material properties

Young modulus: $E = 2.10^5 \text{ MPa}$

Poisson's ratio: $\nu = 0.3$

Density: $\rho = 7850 \text{ kg/m}^3$

1.3 Boundary conditions and loadings

Lower face : uniform constraint of traction $\sigma_z = 1. \text{ MPa}$

Higher face : uniform constraint of traction $\sigma_z = 1. \text{ MPa}$

According to modeling, one also has boundary conditions of symmetry and blocking of the movements of rigid body.

In the modeling D where only the quarter of the parallelepiped is represented, one uses boundary conditions of antisymmetry for the loading of torsion: they amount imposing worthless tangential displacements on a face. The loading of torsion is introduced in the form of a tangential surface force (shearing distributed) applied to the lips of the crack.

- Upper lip: $F_x = -\tau \frac{Y}{a}$ and $F_y = +\tau \frac{X}{a}$
- Lower lip: $F_x = +\tau \frac{Y}{a}$ and $F_y = -\tau \frac{X}{a}$

2 Reference solution

2.1 Method of calculating used for the reference solution

For a circular crack of ray a in an infinite medium, subjected to a uniform traction σ according to the normal with the plan of the lips, the rate of refund of energy room $G(s)$ is independent of the curvilinear X-coordinate s and is worth [bib1]:

$$G(s) = \frac{(1-\nu^2)}{\pi E} 4\sigma^2 a$$

then the coefficient of intensity of constraint K_I is given by the formula of Irwin:

$$G(s) = \frac{(1-\nu^2)}{E} K_I^2 \text{ that is to say } K_I = \frac{2\sigma\sqrt{a}}{\sqrt{\pi}}$$

If this crack is subjected to a shearing distributed on the lips: $\sigma_{\theta z} = \tau \frac{r}{a}$

(what is equivalent to a torsion ad infinitum), then one is in pure mode 3 and the stress intensity factor corresponding is worth:

$$K_3 = \frac{4\tau\sqrt{a}}{3\sqrt{\pi}} \text{ thus by the formula of Irwin } G(s) = \frac{(1+\nu)}{E} K_3^2$$

In the presence of the two combined modes, one will have:

$$G(s) = \frac{(1-\nu^2)}{E} K_I^2 + \frac{(1+\nu)}{E} K_3^2$$

2.2 Results of reference

Digital application (case with loading of traction only):

For the loading considered and $a = 2m$, one obtains then:

$$G(s) = 11.586 J/m^2$$
$$KI = 1.5958E6 J/m^2$$

For modeling G (3 different funds of crack) with the same loading, one obtains:

for $a = 2m$

$$G(s) = 10.586 J/m^2$$
$$KI = 1.5958E6 J/m^2$$

for $a = 1.88m$

$$G(s) = 10.891 J/m^2$$
$$KI = 1.5472E6 J/m^2$$

for $a = 1.76m$

$$G(s) = 10.196 J/m^2$$
$$KI = 1.4969E6 J/m^2$$

Digital application (case with loading of torsion only):

$$G(s) = 7.3565 J/m^2$$
$$KI = 1.0638E6 J/m^2$$

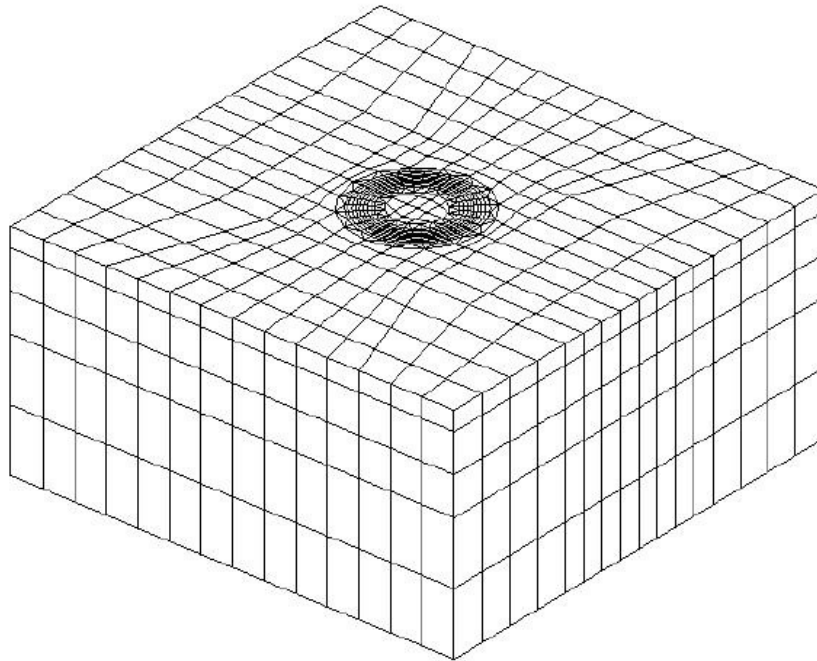
2.3 References bibliographical

- 1) Solution of Sneddon (1946) in G.C. Sih: Handbook of stress-intensity factors Institute of Fracture and Solid Mechanics - Lehigh University Bethlehem, Pennsylvania

3 Modeling A

The bottom of crack is closed. One calculates G .

3.1 Characteristics of modeling



The interest of this modeling is to represent the entirety of the bottom of crack which is a closed curve, without benefitting from symmetries of the problem.

Only the loading of traction is taken into account.

3.2 Characteristics of the grid

Many nodes: 11114

Number of meshes and type: 2432 PENTA15

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges.

3.3 Notice

The keyword is used `SYME` in the operator `CALC_G` to multiply automatically by two the rate of refund of energy calculated on only one lip of the crack, when the operand `FOND_FISS` is absent. When `FOND_FISS` is present, information on symmetry is recovered directly in the concept `fond_fiss` created via `DEFI_FOND_FISS`.

The principle is the same one for `POST_K1_K2_K3`. The indication of symmetry induced the calculation of the factors of intensity of the constraints and the rate of refund of energy `G_IRWIN` by interpolation of displacements of a single lip of the crack. The displacement of the nodes mediums of

the edges of elements touching the bottom of crack to the quarter of these edges would make it possible to improve the precision of calculation.

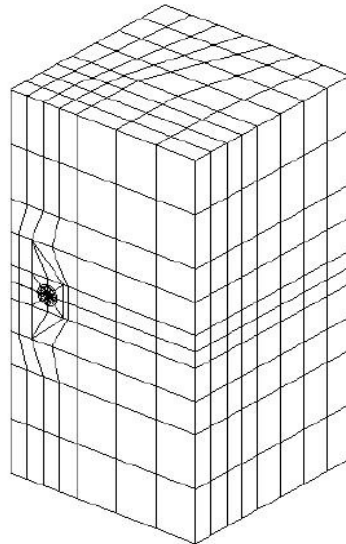
3.4 Sizes tested and results

Identification	Reference	Type of reference	% tolerance
G local Node <i>N403</i> - G Lagrange	11,586	ANALYTICAL	3,0
G local Node <i>N2862</i> - G Lagrange	11,586	ANALYTICAL	2,0
G local Node <i>N375</i> - G Lagrange	11,586	ANALYTICAL	3,0
G local Node <i>N292</i> - G Lagrange	11,586	ANALYTICAL	2.4
$\max(G_{local})$ - G Lagrange	11,59	ANALYTICAL	2.5
$\min(G_{local})$ - G Lagrange	11,59	ANALYTICAL	2,0
G local Node <i>N403</i> - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node <i>N2862</i> - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node <i>N375</i> - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local not 1 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 13 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 21 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local not 33 LAGRANGE / NB_POINT_FOND=33	11,586	ANALYTICAL	1,0
G local Node <i>N292</i> - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G (POST_K1_K2_K3) - Node <i>N403</i>	11,586	ANALYTICAL	5,0
G (POST_K1_K2_K3) - Node <i>N2862</i>	11,586	ANALYTICAL	10,0
G (POST_K1_K2_K3) - Node <i>N375</i>	11,586	ANALYTICAL	5,0
G (POST_K1_K2_K3) - Node <i>N292</i>	11,586	ANALYTICAL	5,0

4 Modeling B

Calculation with POST_K1_K2_K3 in 3D

4.1 Characteristics of modeling



This modeling makes it possible to test the calculation of K_I using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack). The parameter ABSC_CURV_MAXI of the operator is calculated in POST_K1_K2_K3 so as to retain 5 nodes on the segment of extrapolation ($d_{max} = 0,35$).

Only the loading of traction is taken into account.

4.2 Characteristics of the grid

Many nodes: 6536

Number of meshes and type: 432 PENTA15 and 987 HEXA20

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

4.3 Sizes tested and results

4.3.1 Results of CALC_G

Identification	Reference (analytical)	% tolerance
G room Node 49	11,59	3.0
G room Node 1710	11,59	2.0
G room Node 77	11,59	3.0

4.3.2 Results of POST_K1_K2_K3

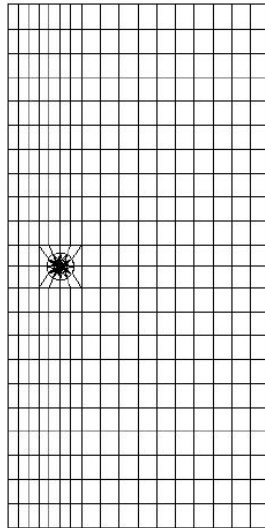
Identification	Reference (analytical)	% tolerance
<i>G</i> Node 49	11,586	2
<i>G</i> Node 77	11,586	2
<i>G</i> Node 1710	11,586	2
<i>KI</i> Node 49	1,60E+006	1
<i>KI</i> Node 77	1,60E+006	1
<i>KI</i> Node 1710	1,60E+006	1

Other purely data-processing tests of the order POST_K1_K2_K3 are also carried out.

5 Modeling C

Calculation with POST_K1_K2_K3 into axisymmetric.

5.1 Characteristics of modeling



00001 00001

This modeling makes it possible to test the calculation of K_I using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack) into axisymmetric.

Only the loading of traction is retained in this modeling.

5.2 Characteristics of the grid

Many nodes: 1477

Number of meshes and type: 402 QUAD8 and 60 TRIA6

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

5.3 Sizes tested and results

Identification	Method	Reference	Type of reference	% tolerance
G	CALC_G	23,2	ANALYTICAL	1,8
K_I	POST_K1_K2_K3	1,60E+006	ANALYTICAL	3
G	POST_K1_K2_K3	11,6	ANALYTICAL	1,8

5.4 Notice

An order of STAT_NON_LINE is added to validate the modifications brought on xpesro.f . The got results this case test will be compared with those obtained from case test sslv134i for X-FEM into

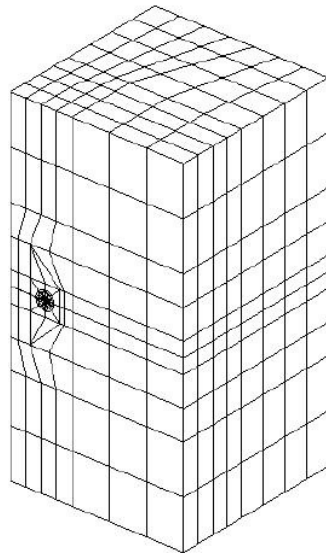
axisymmetric. In addition of the simple traction applied to the edges high and low of the plate, a rotation of 150 trs/min around the symmetrical axis is applied.

With the option `CALC_G`, $G=21,4$ and $K_1=219\text{E}+05$.

6 Modeling D

Calculation with POST_K1_K2_K3 in 3D for modes 1 and 3.

6.1 Characteristics of modeling



The boundary conditions following are successively applied:

- traction: as for modeling B;
- torsion.

This modeling makes it possible to test the calculation of K_I and K_{II} compounds using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of the crack).

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

6.2 Characteristics of the grid

Many nodes: 6536

Number of meshes and type: 432 PENTA15 and 987 HEXA20

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges, to obtain a better precision.

6.3 Notice

The two loading cases (traction and torsion) are taken into account. It is thus necessary to cumulate the values of G for the two loadings.

Thus

$$G(s) = (11.586 + 7.356) = 18.943 \text{ J/m}^2$$

Only traction contributes to K_I , only torsion contributes to K_{II} .

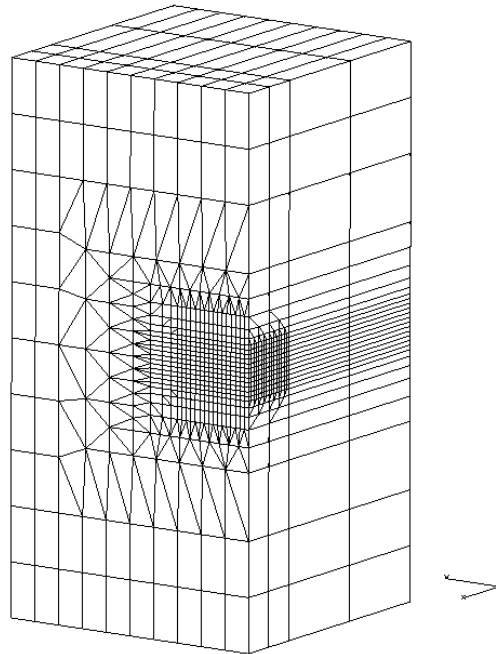
6.4 Sizes tested and results

Identification	Method	Localization	Reference	Type of reference	% tolerance
G	CALC_G Legendre	Node 49	18,94	ANALYTICAL	3,0
G	CALC_G Legendre	Node 1710	18,94	ANALYTICAL	2,0
G	CALC_G Legendre	Node 77	18,94	ANALYTICAL	3,0
K1	POST_K1_K1_K3	Node 49	1.596 10 ⁶	ANALYTICAL	1,0
K1	POST_K1_K1_K3	Node 1710	1.596 10 ⁶	ANALYTICAL	1,0
K1	POST_K1_K1_K3	Node 77	1.596 10 ⁶	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 49	1.064 10 ⁶	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 1710	1.064 10 ⁶	ANALYTICAL	2,0
K3	POST_K1_K1_K3	Node 77	1.064 10 ⁶	ANALYTICAL	1,0
G	POST_K1_K1_K3	Node 49	18,94	ANALYTICAL	2,0
G	POST_K1_K1_K3	Node 1710	18,94	ANALYTICAL	2,0
G	POST_K1_K1_K3	Node 77	18,94	ANALYTICAL	2,0

7 Modeling H

Method X-FEM.

7.1 Characteristics of modeling



This modeling makes it possible to test the calculation of K_I using `POST_K1_K2_K3` and `CALC_G` (option `CALC_K_G`) on a crack nonwith a grid (method `X-FEM`).

Only the loading of traction is taken into account. Conditions of symmetry are imposed on the two side faces. Moreover, one should not forget to impose the conditions of symmetry on the lips of the crack through the degrees of freedom of jump `H1X`, `H1Y` and `H1Z`.

7.2 Characteristics of the grid

Many nodes: 6100

Number of meshes and type: 1500 `PENTA6` and 4600 `HEXA8` (linear grid)

7.3 Sizes tested and results

The values tested are the factors of intensity of the constraints K_I along the bottom of crack, calculated either by `POST_K1_K2_K3` (method 3), that is to say by `CALC_G`. The test is carried out in 3 points of the bottom of crack: items 1 (first point), 10 and 24 (last point).

The average quadratic error corresponds to the following quantity:

$$\varepsilon = \sqrt{\frac{\int_{\Gamma} (K_I^{ref} - K_I^{Aster})^2 ds}{\int_{\Gamma} (K_I^{ref})^2 ds}}$$

Identification	Reference	Type of reference	% tolerance
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CALC_G			
<i>KI</i> - point 1	1.595e6	ANALYTICAL	12,00
<i>KI</i> - point 10	1.595e6	ANALYTICAL	3,00
<i>KI</i> - point 24	1.595e6	ANALYTICAL	18,00
Average quadratic error			9,90

POST_K1_K2_K3			
<i>KI</i> - point 1	1.595e6	ANALYTICAL	2,00
<i>KI</i> - point 10	1.595e6	ANALYTICAL	2,00
<i>KI</i> - point 24	1.595e6	ANALYTICAL	2,00
Average quadratic error			1,15

Remarks

Precision of the results got on a crack nonwith a grid (method X-FEM) and postprocessing POST_K1_K2_K3 is very satisfactory, comparable to the precision with a crack with a grid FEM. Nevertheless, the results are degraded with CALC_G as it is generally the case with the curved faces (test SSVL154 also). The use of this operator is to be disadvised for the nonright faces with the method X-FEM, being given the rather modest precision of this operator (18%).

For the operator CALC_G, smoothings of the type LAGRANGE do not allow to have easily useable results; a smoothing of the type LEGENDRE is thus to privilege.

It is pointed out that the grid used is linear; the use of a finer grid makes it possible to improve the precision of the result, but to the detriment of the computing times.

8 Modeling I

Method X-FEM into axisymmetric.

8.1 Characteristics of modeling

This modeling makes it possible to test the calculation of K_I using `POST_K1_K2_K3` and `CALC_G` (option `CALC_K_G`) on an axisymmetric crack nonwith a grid (method X-FEM).

Two types of loadings are considered. First is of simple traction applied to the edges high and Bace of the plate. Second is of simple traction applied to the edges high and low of the plate and a rotation of 150 trs/min around the symmetrical axis.

8.2 Characteristics of the grid

Many nodes: 20301

Number of meshes and type: 20000 `QUA4` and 600 `SE2` (linear grid)

8.3 Sizes tested and results

One tests the values of G and K_I calculated by the order `CALC_G` option '`CALC_K_G`' and by the order `POST_K1_K2_K3`, as well as the .valor of G calculated by the order `CALC_G` (option '`CALC_G`').

Loading 1: simple traction applied to the edges high and Bace of the plate

Identification	Type of reference	Value of reference	Tolerance (%)
G (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'ANALYTICAL'	11.59	2.1%
K_I (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'ANALYTICAL'	1,60E+06	6.0%
G (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'AUTRE_ASTER'	11.78	0.4%
K_I (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'AUTRE_ASTER'	1,64E+06	2.0%
G (<code>CALC_G</code> option ' <code>CALC_G</code> ')	'ANALYTICAL'	23.17	2.1%
G (<code>POST_K1_K2_K3</code>)	'ANALYTICAL'	11.59	6.0%
K_I (<code>POST_K1_K2_K3</code>)	'ANALYTICAL'	1,60E+06	6.0%

Loading 2: simple traction applied to the edges high and low of the plate and a rotation of 150 trs/min around the symmetrical axis_

Identification	Type of reference	Value of reference	Tolerance (%)
G (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'AUTRE_ASTER'	2136.52	0.3%
K_I (<code>CALC_G</code> option ' <code>CALC_K_G</code> ')	'AUTRE_ASTER'	2,191E+07	2.5%
G (<code>CALC_G</code> option ' <code>CALC_G</code> ')	'AUTRE_ASTER'	4273.04	0.3%

G (POST_K1_K2_K3)	'AUTRE_ASTER'	2136.52	4.0%
KI (POST_K1_K2_K3)	'AUTRE_ASTER'	2,191E+07	2.5%

8.4 Remarks

In this case test, the ratio a/W between the size of the crack a and the width W is 0,2 . The effects edges thus contribute to the difference between the digital solution for a finished edge and the reference solution for a continuous medium.

9 Modeling J

Fund of crack closed, calculation of G for the incompressible elements.

9.1 Characteristics of modeling

Identical to modeling A except the elements used are 3D_INCO_UPG and 3D_INCO_UP.

9.2 Characteristics of the grid

Identical to modeling A

9.3 Notice

The keyword is used SYME in the operator CALC_G to multiply automatically by two the rate of refund of energy calculated on only one lip of the crack, when the operand FOND_FISS is absent. When FOND_FISS is present, information on symmetry is recovered directly in the concept fond_fiss created via DEFI_FOND_FISS.

9.4 Sizes tested and results

Identification	Reference	Type of reference	% tolerance
G local Node <i>N403</i> - G Lagrange	11,586	ANALYTICAL	4.0
G local Node <i>N2862</i> - G Lagrange	11,586	ANALYTICAL	1.0
G local Node <i>N375</i> - G Lagrange	11,586	ANALYTICAL	3.5
G local Node <i>N292</i> - G Lagrange	11,586	ANALYTICAL	4.0
<i>max(G local)</i> - G Lagrange	11,590	ANALYTICAL	5.0
<i>min(G local)</i> - G Lagrange	11,586	ANALYTICAL	1.0
G local Node <i>N403</i> - G Lagrange_no_no	11,586	ANALYTICAL	2.5
G local Node <i>N2862</i> - G Lagrange_no_no	11,586	ANALYTICAL	2,0
G local Node <i>N375</i> - G Lagrange_no_no	11,586	ANALYTICAL	2.0
G local Node <i>N292</i> - G Lagrange_no_no	11,586	ANALYTICAL	3.0
<i>max(G local)</i> - G Lagrange_no_no	11,715	ANALYTICAL	2.0
<i>min(G local)</i> - G Lagrange_no_no	11,575	ANALYTICAL	2.0

10 Modeling K

One tests K into axisymmetric for the incompressible elements

10.1 Characteristics of modeling

Identical to modeling C except the elements used are `AXIS _INCO_UPG` and `AXIS _INCO_UP`.

10.2 Characteristics of the grid

Identical to modeling C

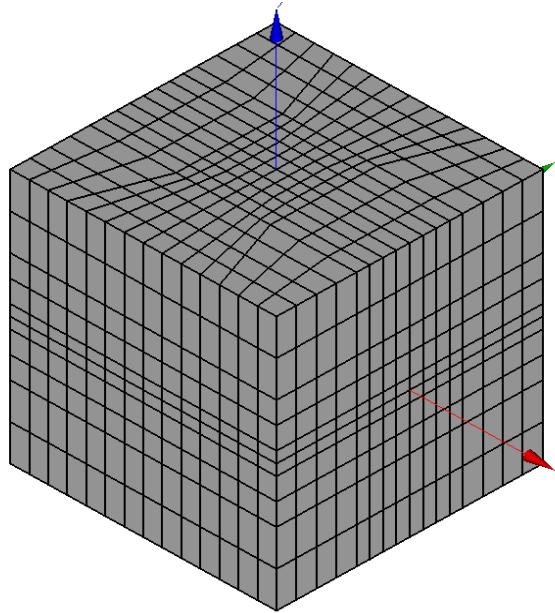
10.3 Sizes tested and results

Identification	Method	Reference	Type of reference	% tolerance
G	CALC_G	23,2	ANALYTICAL	2
G	CALC_K_G	23,2	ANALYTICAL	2
KI	CALC_K_G	1.595769E6	ANALYTICAL	3.5
KI	CALC_K_G	1.643E+06	ANALYTICAL	3

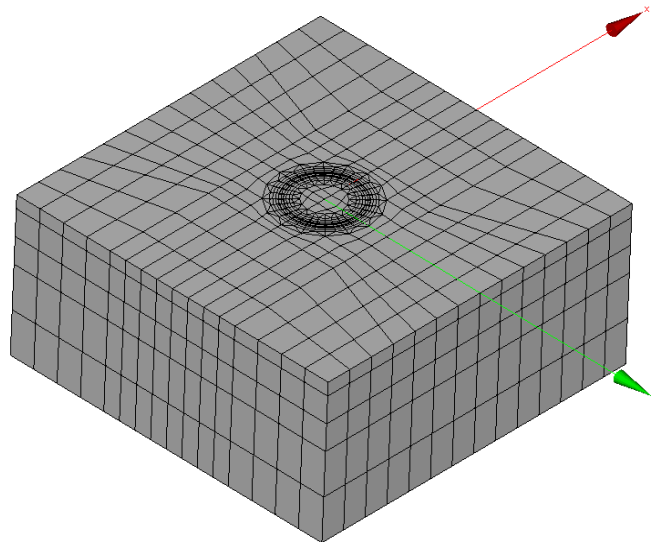
11 Modeling L

The bottom of crack is closed. One calculates G .

11.1 Characteristics of modeling



The interest of this modeling is to set out again of modeling A and to symmetrize the grid.



Thus, we find the discretization of the crown of modeling A.

Only the loading of traction is taken into account.

11.2 Characteristics of the grid

Many nodes: 18571

Number of meshes and type: 3136 HEXA20 and 1728 PENTA15

11.3 Sizes tested and results

Identification	Reference	Type of reference	% tolerance
G local not 3 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	5, 0
G local not 8 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	1, 0
G local not 11 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	4, 0
G local not 15 LAGRANGE / NB_POINT_FOND=16	11,586	ANALYTICAL	2, 0

12 Summaries of the results

The conclusions of this case test are the following ones:

- The definition and the calculation of G room on closed funds of crack is validated. One checks in particular the independence of G room with respect to the angle for an axisymmetric crack and a loading. One notes a variation of less 2% on the whole of the bottom of crack by the methods 'LAGRANGE' and 'LAGRANGE_NO_NO'.
- The order POST_K1_K2_K3, which makes it possible to calculate the stress intensity factors by exploiting the jump of displacements on the lips of the crack, is also validated. This method, less precise than CALC_G, allows to obtain here (with a suitable grid: nodes mediums of the edges touching the bottom of crack moved with the quarter of these edges) of the values of $K1$ and $K3$ with less 2% reference.

Three methods of interpolation are used and give close results. Method 3 is interesting because it provides a single value of the stress intensity factors and not a maximum value and a minimal value.

The use of POST_K1_K2_K3 to study a crack by relaxation of nodes is tested and gives satisfactory results.

- One validates the calculation of the bilinear form of G and options G_MAX and CALC_K_MAX.
- Method X-FEM allows to evaluate the factors of intensity of the constraints K on a grid not fissured with an error lower than 20% with CALC_G. With operator POST_K1_K2_K3, the precision increases considerably, to reach a variation with the analytical solution of 2%.
- One validates calculations for elements 3D_INCO_UPG, 3D_INCO_UP, AXIS_INCO_UPG and AXIS_INCO_UP