SSLV135 – Criteria of starting in fatigue under multiaxial loadings for a structure

Summary:

This CAS-test aims to test the operator `CALC_FATIGUE` who calculates the damage for all the points of the struCture.

The geometry treated here is a cube without defect with which one carries out a linear elastic mechanical calculation followed by the calculation of the critical plan of shearing in each point of Gauss and each node.

- modeling a: criteria `MATAKE_MODI_AC`, `DANG_VAN_MODI_AC`, `VMIS_TRESCA`, criterion in formula, periodic and biaxial loading proportional;
- modeling b: criterion `MATAKE_MODI_AV`, `DANG_VAN_MODI_AV`, `FATESOCI_MODI_AV`, criterion in formula, loading not-periodical and biaxial proportional;
- modeling C: criteria `MATAKE_MODI_AC` and `DANG_VAN_MODI_AV`, criterion in formula, loading multiaxial, non-proportional;
- modeling D : criterion in formulE, to test new sizes, tankgemmate uniaxial elastoplastic and periodic.
- modeling E : criterion in formulE, to test new sizes (in constraints) , tankgemmate biaxial elastoplastic and not-periodical.
- modeling F : criterion in formulE, to test new sizes (in deformations) , tankgemmate biaxial rubber band with variable amplitude and tankgemmate uniaxial elastoplastic and not-periodical.
- modeling G: criteria `MATAKE_MODI_AC`, `DANG_VAN_MODI_AC`, `MATAKE_MODI_AV`, `DANG_VAN_MODI_AV`, `FATESOCI_MODI_AV`. One tests the change of the direction of the critical plan on which the damage or shearing is maximum.
- modeling H: criteria in formula of the standard plan criticizes with the keyword `FORMULE_CRITIQUE`
- modeling I : criterion in formula, identical to modeling F (elastic biaxial loading with variable amplitude) .

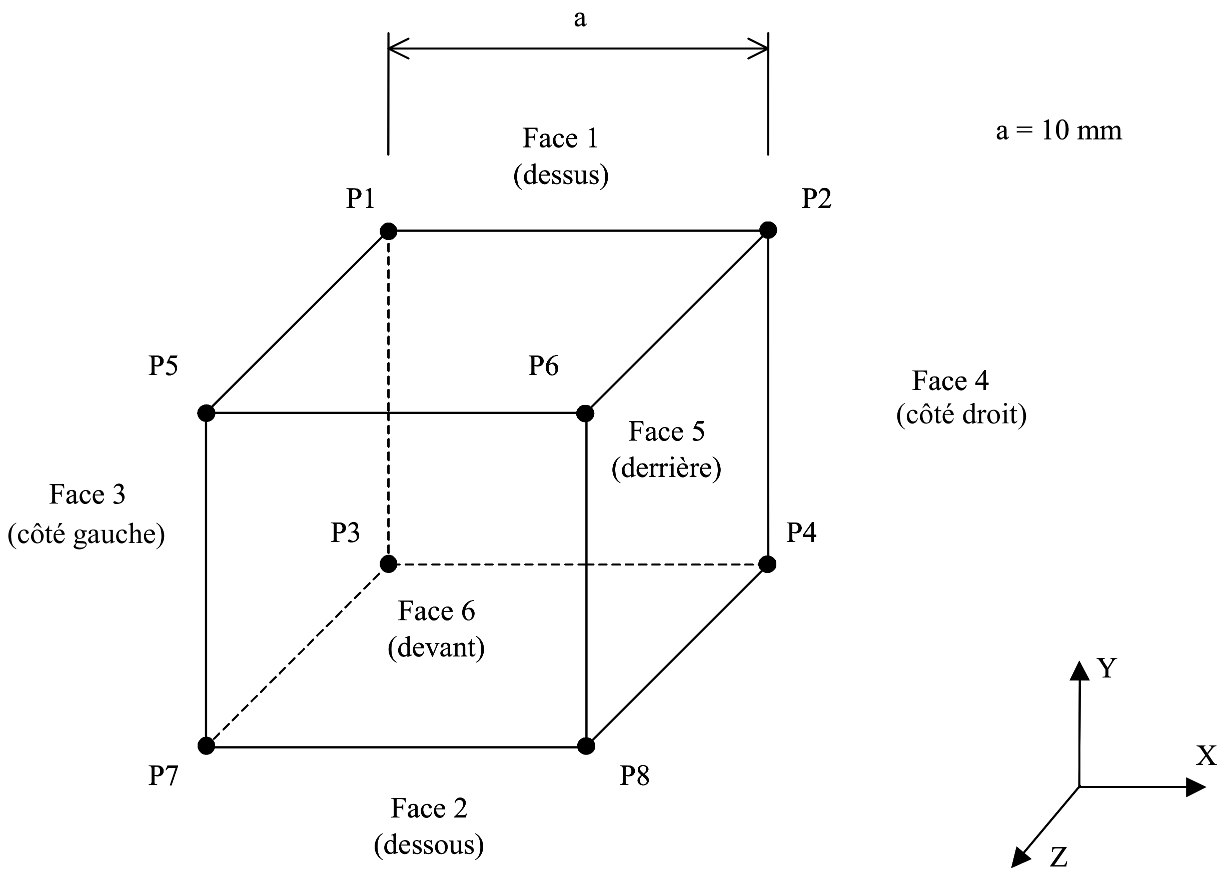
The criteria in modeling A are said “to plan of critical shearing”, they are adapted to the periodic loadings. The criteria in modeling B can be qualified criteria “with plan of critical damage”, they can be used when the loading is not periodical. The criterion `VMIS_TRESCA` is not a criterion of tiredness, it determines the variation of maximum amplitude of the tensor of the constraints. The criteria formulates some making it possible to the user to build criteria

according to the predefined sizes are also tested. The criteria in modeling H are said “to plan criticizes general”, they are adapted to the periodic loadings. Modeling I takes again the elastic case of modeling F.

Note: modelings B, E and F are in the base of validation and not in the base of checking. They are thus not accessible outside EDF.

1 Problem of reference

1.1 Geometry



The cube has $10\,mm$ on side.

1.2 Material properties

1.2.1 Modelings: WITH, B, C, F and I

Young modulus: $E=200000\,MPa$
Poisson's ratio: $\nu=0.3$

1.2.2 Modelings D and H

modulus Young and the Poisson's ratio is identical to those of other modelings.
Yield stress of material: $\sigma_o=150.0\ MPa$
The elastoplastic behavior of Von Mises with linear isotropic work hardening with the slope of curve of work hardening: $H=50000.0\ MPa$

1.2.3 Modeling E

modulus Young and the Poisson's ratio is identical to those of other modelings.
Yield stress of material: $\sigma_o=900.0\ MPa$

The elastoplastic behavior of Von Mises with linear isotropic work hardening with the slope of curve of work hardening: $H=50000.0\ MPa$

1.2.4 Modeling F and I

In this modeling, two different materials are considered. First is similar to paragraph 1.2.1 (i.e. purely elastic), and second is similar to material used in modeling E (1.2.3).

1.2.5 Modeling G

In this modeling, two different materials are considered.
The first material is elastic with the Young modulus: $E=193000\ MPa$ and the Poisson's ratio: $\nu=0.3$
The second material is elastoplastic, the modulus Young and the Poisson's ratio is identical to first material. Yield stress of material: $\sigma_o=208.0\ MPa$. The elastoplastic behavior of Von Mises with linear isotropic work hardening with the slope of curve of work hardening: $H=50000.0\ MPa$

1.3 Curves of Wöhler and Manson-Whetstone sheath

Modelings use all the curves of Wöhler (i.e. a curve in constraint) and of Manson-Whetstone sheath (i.e. a curve in deformation).

Here the curve of Wöhler (alternate traction and compression controlled in constraint):

Half amplitude of constraint (MPa)	138.0	152.0	165.0	180.0	200.0	250.0	295.0
Many cycles	1.0E+6	5.0E+5	2.0E+5	1.0E+5	5.0E+4	2.0E+4	1.2E+4
Half amplitude of constraint (MPa)	305.0	340.0	430.0	540.0	690.0	930.0	1210.0
Many cycles	1.0E+4	5.0E+3	2.0E+3	1.0E+3	5.0E+2	2.0E+2	1.0E+2
Half amplitude of constraint (MPa)	1590.0	2210.0	2900.0				
Many cycles	5.0E+1	2.0E+1	1.0E+1				

Table 1.3-1: Curve of Wöhler

Here the curve of Manson-Whetstone sheath (alternate traction and compression controlled in deformation):

Deformation	0.00226	0.0023	0.0025	0.0027	0,003	0.0035
Many cycles	5.8E+6	4.6E+6	2.39284E+5	1.49535E+5	7.3544E+4	3.3821E+4
Deformation	0,006	0.0085	0,010	1,000		
Many cycles	2.85E+3	1.068E+3	5.62E+2	1.0		

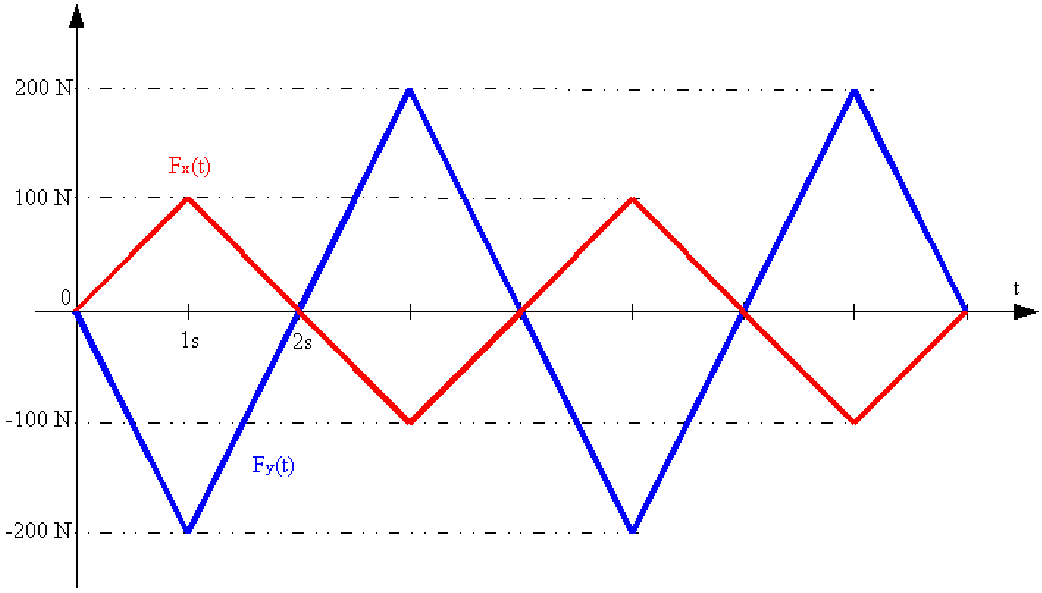
Table 1.3-2: Curve of Manson-Whetstone sheath

1.4 Boundary conditions and loadings

1.4.1 Modelings: WITH, B

- Displacements according to the axis X face 3 are blocked ($DX=0.0$).
- Displacements according to the axis Y face 2 are blocked ($DY=0.0$).
- Displacements of the point $P3$ are blocked according to the axis Z ($DZ=0.0$).
- We apply an alternate biaxial loading (traction and compression) according to the axes X and Y. $Fx(t)$ represent the alternate efforts applied to face 4 according to the axis X and $Fy(t)$ represent the alternate efforts applied to the face 1 according to the axis Y.

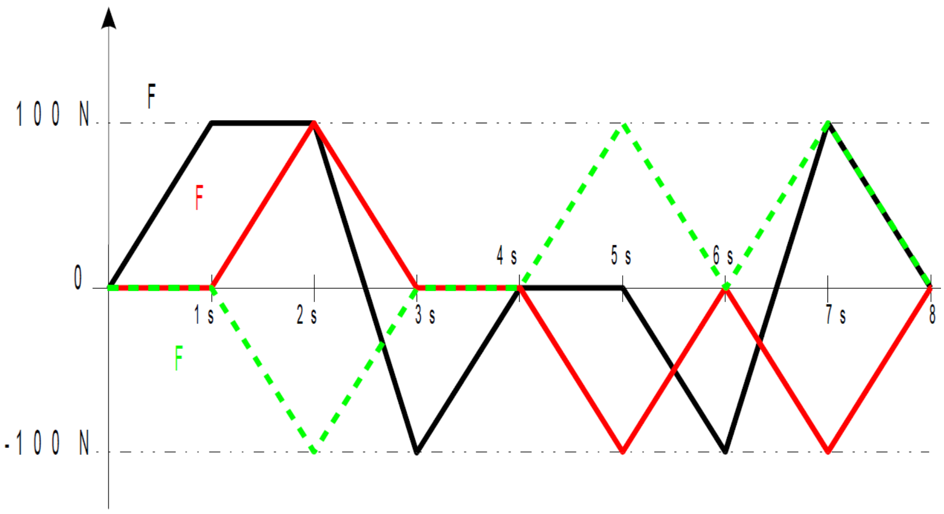
Loading for these modelings:



1.4.2 Modeling C

- Displacements according to the axis X face 3 are blocked ($DX=0.0$).
- Displacements according to the axis Y face 3 are blocked ($DY=0.0$).
- Displacements according to the axis Z face 2 are blocked ($DZ=0.0$).
- We apply a multiaxial loading: traction and compression according to the axis X, shearing according to the axes Y and Z. $Fx(t)$ represent the efforts applied to face 4 according to the axis X, $Fy(t)$ represent the efforts applied to face 4 according to the axis Y and $Fz(t)$ represent the efforts applied to face 1 according to the axis Z.

Loading for this modeling:



1.4.3 Modelings D and H

- Displacements according to the axis X face 3 are blocked ($DX=0.0$).
- Displacements according to the axis Y face 3 are blocked ($DY=0.0$).

- Displacements according to the axis Z face 2 are blocked ($DZ=0.0$).
- In this modeling, we apply a periodic uniaxial loading.

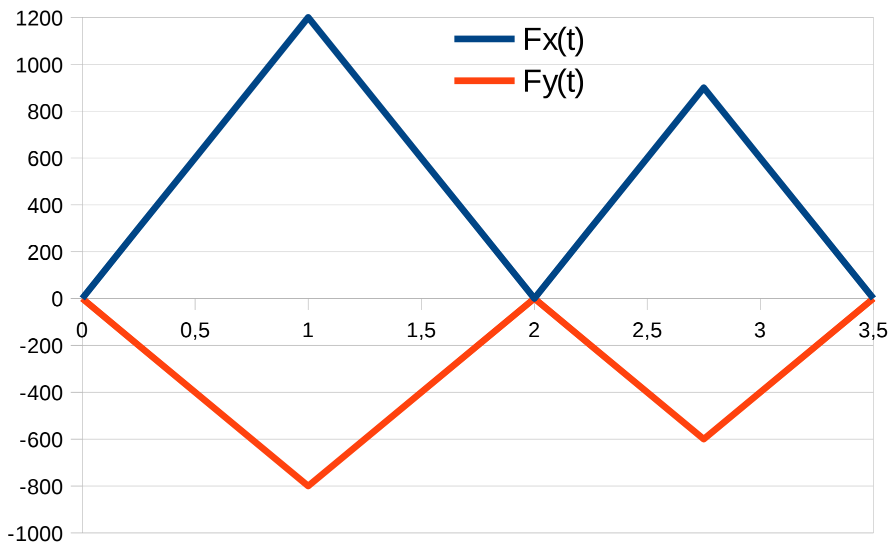
t	0	1	2	3	4	5	6
$Fx(t)$	0	200	0	-200	0	200	0

It is noted that this history of loading involves a plastic deformation in calculation. It is also noted that the monotonous history of loading enters $t=0$ and $t=1$ (the part of the monotonous loading) is not taken into account in the fatigue analysis.

1.4.4 Modeling E

- Displacements blocked here are the same ones as in modeling O
- We apply a biaxial loading here not-periodical.

t	0	1	2	2,75	3,5
$Fx(t)$	0	1200	0	900	0
$Fy(t)$	0	-800	0	-600	0

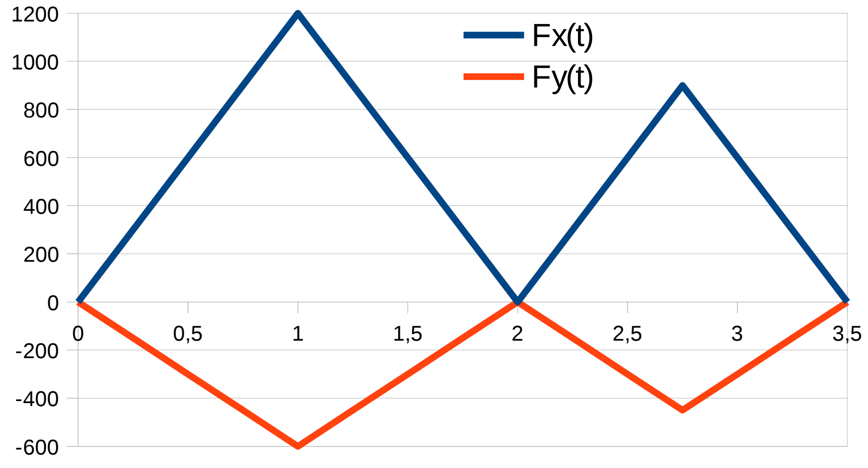


1.4.5 Modeling F and I

- Displacements blocked here are the same ones as in modeling O
- In this modeling, we apply three distinct loadings. First is biaxial, the two others are uniaxial.

First loading (association of `TR_CS` and of `COEFF` in the command file):

t	0	1	2	2,75	3,5
$Fx(t)$	0	1200	0	900	0
$Fy(t)$	0	-600	0	-450	0



Second loading (association of `TR_CS2` and of `COEF2` in the command file):

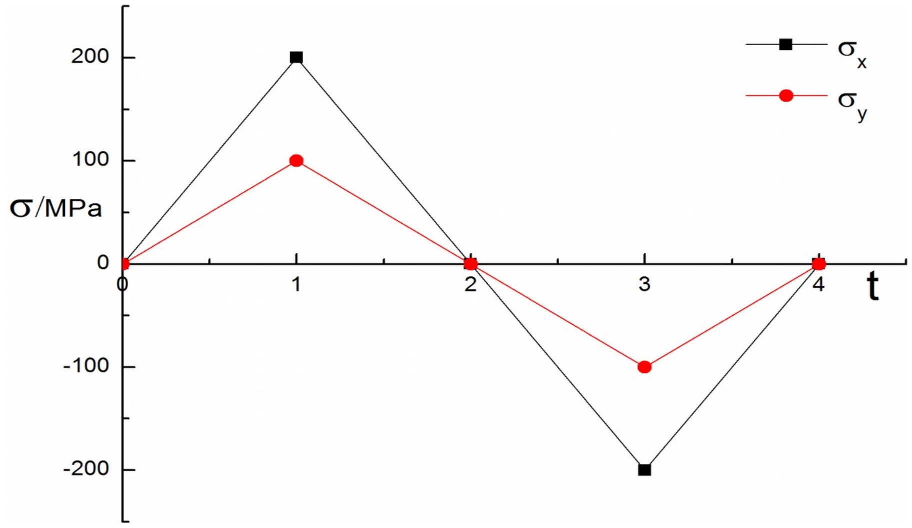
t	0	1	2.5625	3.5625	5.5625
$Fx(t)$	0	1200	-675	900	-675

Third loading (association of `TR_CS2` and of `COEF3` in the command file):

t	0	1	2	3	4
$Fx(t)$	0	1800	-600	2400	-1200

1.4.6 Modeling G

- Displacements blocked here are the same ones as in modeling A
- We apply an alternate biaxial loading (traction and compression) according to the axes X and Y. σ_x represent the alternate efforts applied to face 4 according to the axis X and σ_y represent the alternate efforts applied to the face 1 according to the axis Y.
- To study effects of the average constraint on the orientation of the critical plan, only of the loadings biaxial, proportional according to directions of X and Y are considered.



- Two parameters are defined

$$\lambda=\frac{\sigma_{y,a}}{\sigma_{x,a}}$$

$$\alpha=\frac{\sigma_{y,m}}{\sigma_{y,a}}=\frac{\sigma_{x,m}}{\sigma_{x,a}}$$

where $\sigma_{x,a}$, $\sigma_{y,a}$ amplitudes of the constraints represent according to X and Y, respectively. $\sigma_{x,m}$ and $\sigma_{y,m}$ the values of the average constraints represent according to X and Y directions, respectively. One takes $\sigma_{x,a}=200\,MPa$, $\lambda=1$ and α vary from -1 to 10 with an interval of 0.5.

1.5 Initial conditions

Without object for a static analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

In the case of an alternate biaxial loading where the pressures applied are such as: $\sigma_x = \lambda\,\sigma_y$, with $|\lambda| > 1$ and $\lambda < 0$, it is shown [bib1] that the half amplitude of maximum shearing $\Delta\tau/2 = (\Delta\sigma_x + \Delta\sigma_y)/4$, where $\Delta\sigma_x/2$ and $\Delta\sigma_y/2$ the half amplitudes of pressures applied according to the axes represent x and y . Moreover, there are two critical plans in which shearing is maximum:

2.2 Results of reference for modeling A

See the references [bib1] and [R7.04.04].

Half amplitude of maximum shearing:
N 1 **N** 2

$\Delta\sigma_x/2$ (MPa)	$\Delta\sigma_y/2$ (MPa)	$\Delta\tau/2$ (MPa)
100	200	150

Note:

The half amplitude of maximum shearing is identical for the two critical plans.

Normal vectors with the two critical plans:

	n_1	n_2
Component x	$-1/\sqrt{2}$	$1/\sqrt{2}$
Component y	$1/\sqrt{2}$	$1/\sqrt{2}$
Component z	0	0

Normal maximum constraints in the fields of the normals n_1 and n_2 :

$N_{\max}(n_1) = 50\ MPa$ and $N_{\max}(n_2) = 50\ MPa$.

Hydrostatic pressure maximum, independent with respect to the plans of normals n_1 and n_2 :

$P = 33.33333\ MPa$.

Normal average constraints in the fields of the normals n_1 and n_2 :

$N_m(n_1) = 0\ MPa$ and $N_m(n_2) = 0\ MPa$.

Normal maximum deformations in the fields of the normals n_1 and n_2 :

$\varepsilon_{\max}(n_1) = 1.75\ 10^{-4}$ and $\varepsilon_{\max}(n_2) = 1.75\ 10^{-4}$

Normal average deformations in the fields of the normals n_1 and n_2 :

$\varepsilon_m(n_1) = 0$ and $\varepsilon_m(n_2) = 0$.

Criterion `MATAKE_MODI_AC`

$$\frac{\Delta\tau(n_i)}{2} + a\,N_{\max}(n_i) \le b\ ,\quad i = 1, 2$$

where $a=1$ and $b=2$.

Equivalent constraints within the meaning of MATAKE in the fields of the normals n_1 and n_2 :

$$\sigma_{eq}(n_i)=\left(\frac{\Delta\tau(n_i)}{2}+a\ N_{\max}(n_i)\right)\frac{f}{t}\ ,\quad i=1,\ 2$$

where f and t represent, respectively, the limit of endurance in alternating bending and the limit of endurance in alternate torsion. Here f/t is equal to $1,5$. Consequently we have:

$$\sigma_{eq}(n_1)=300\ MPa \text{ and } \sigma_{eq}(n_2)=300\ MPa\ .$$

Many cycles to the rupture in the fields of the normals n_1 and n_2 :

Starting from the curve of Wöhler, cf [Table 1.2-1], and from the equivalent constraints within the meaning of MATAKE, we obtain:

$$Nb_{cr}(n_1)=Nb_{cr}(n_2)=10946\ cycles\ .$$

Damage in the fields of the normals n_1 and n_2 :

$$ENDO(n_1)=ENDO(n_2)=9.13565\ 10^{-5}\ .$$

Criterion of Dang Van adapted to the periodic loadings : DANG_VAN_MODI_AC

$$\frac{\Delta\tau(n_i)}{2}+a\ P\leq b\ ,\quad i=1,\ 2$$

where $a=1$ and $b=2$.

Equivalent constraints within the meaning of DANG VAN in the fields of the normals $\mathbf{n}_1$ and $\mathbf{n}_2$:

$$\sigma_{eq}(n_i)=\left(\frac{\Delta\tau(n_i)}{2}+a\ P\right)\frac{c}{t}\ ,\quad i=1,\ 2$$

where c and t represent, respectively, the limit of endurance in alternate shearing and the limit of endurance in alternate traction and compression. Here c/t is equal to $1,5$. Consequently we have:

$$\sigma_{eq}(n_1)=275\ MPa \text{ and } \sigma_{eq}(n_2)=275\ MPa\ .$$

Many cycles to the rupture in the fields of the normals n_1 and n_2

Starting from the curve of Wöhler, cf [Table 1.2-1], and from the equivalent constraints within the meaning of DANG VAN, we obtain:

$$Nb_{cr}(n_1)=Nb_{cr}(n_2)=14903\ cycles\ .$$

Damage in the fields of the normals n_1 and n_2 :

$$ENDO(n_1)=ENDO(n_2)=6.709959\ 10^{-5}\ .$$

For the option COURBE_GRD_VIE = 'FORMES_VIE' and FORMULE_VIE = WHOL_F where the curve of life is provided by a formula, the results of reference are identical to those called with the names except those of NBRUPT and ENDO as one uses a curve of different life.

- the curve WHOL_F of formula (grandeur_équivalente = $4098.3\times(NBRUP^{-0.2693})$) is initially provided by a function tabulée to calculate values of reference of NBRUPT and ENDO.

- the curve MANCO2 of formula (grandeur_équivalente = $0.2\times(NBRUP^{-0.1619})$) is initially provided by a function tabulée to calculate values of reference of NBRUPT and ENDO.

2.3 Results of reference for modeling B

Criterion of MATAKE adapted to the nonperiodic loadings: MATAKE_MODI_AV
For this criterion there are no analytical results.

Criterion of Dang Van adapted to the nonperiodic loadings: DANG_VAN_MODI_AV
For this criterion there are no analytical results.
See the references [bib2] and [R7.04.04].

Half amplitude of constraint:

$\Delta\sigma_x/2$ (MPa)	$\Delta\sigma_y/2$ (MPa)
100	200

Criterion of FATEMI and SOCIE adapted to the nonperiodic loadings: FATESOCI_MODI_AV
For this criterion there are no analytical results.
See the references [bib3] and [R7.04.04].

Half amplitude of constraint:

$\Delta\sigma_x/2$ (MPa)	$\Delta\sigma_y/2$ (MPa)
100	200

Criterion of Von Mises and TRESCA applied in search of the maximum variation of a tensor of constraint. The loading can be periodic or not: VMIS_TRESCA

See the reference [R7.04.04].

Half amplitude of constraint:

$\Delta\sigma_x/2$ (MPa)	$\Delta\sigma_y/2$ (MPa)
100	200

Criterion of Von Mises:

Within the space of constraints with six dimensions:

$$\sigma_s=\sqrt{\frac{1}{2}\left[\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+\left(\sigma_{zz}-\sigma_{xx}\right)^2+6\left(\sigma_{xy}^2+\sigma_{xz}^2+\sigma_{yz}^2\right)\right]},$$

with $\sigma_{xx}=200\ MPa$, $\sigma_{yy}=-400\ MPa$ and $\sigma_{zz}=\sigma_{xy}=\sigma_{xz}=\sigma_{yz}=0$, one obtains:

σ_s (MPa)
529.15026221292

Criterion of Tresca:

Within the space of principal constraints with three dimensions:

$$\sigma_s = \mathop{Sup}_{i \neq j} \left(|\sigma_i - \sigma_j| \right),$$

with $\sigma_{xx} = 200\ MPa$, $\sigma_{yy} = -400\ MPa$ and $\sigma_{zz} = \sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0$, one obtains:

σ_s (MPa)
600

2.4 Results of reference for modeling C

There is no result of reference for modeling C, only of the tests of not-regression This modeling has as single objective of to cover the source code.

2.5 Results of reference for modeling D

One starts with a calculation of the constraint, the total deflection and the plastic deformation with the order `CALC_CHAMP`. The results are listed in the following Table by knowing that the other components are equal to zero.

The results are got with the node $N1$.

t	σ_{xx}	ϵ_{xx}^{tot}	ϵ_{yy}^{tot}	ϵ_{zz}^{tot}	ϵ_{xx}^{p}	ϵ_{yy}^{p}	ϵ_{zz}^{p}
0.00E+00	*0.00E+00*	*0.00E+00*	*0.00E+00*	*0.00E+00*	*0.00E+00*	*0.00E+00*	*0.00E+00*
2.50E-01	*5.00E+01*	*2.50E-04*	*-7.50E-05*	*-7.50E-05*	*-1.48E-20*	*-7.40E-20*	*-5.98E-20*
5.00E-01	*1.00E+02*	*5.00E-04*	*-1.50E-04*	*-1.50E-04*	*-4.57E-19*	*2.71E-19*	*2.84E-20*
7.50E-01	*1.50E+02*	*7.50E-04*	*-2.25E-04*	*-2.25E-04*	*-2.29E-19*	*2.15E-19*	*-2.71E-20*
1.00E+00	2.00E+02	1.75E-03	-6.75E-04	-6.75E-04	7.50E-04	-3.75E-04	-3.75E-04
1.25E+00	1.50E+02	1.50E-03	-6.00E-04	-6.00E-04	7.50E-04	-3.75E-04	-3.75E-04
1.50E+00	1.00E+02	1.25E-03	-5.25E-04	-5.25E-04	7.50E-04	-3.75E-04	-3.75E-04
1.75E+00	5.00E+01	1.00E-03	-4.50E-04	-4.50E-04	7.50E-04	-3.75E-04	-3.75E-04
2.00E+00	1.98E-08	7.50E-04	-3.75E-04	-3.75E-04	7.50E-04	-3.75E-04	-3.75E-04
2.25E+00	-5.00E+01	5.00E-04	-3.00E-04	-3.00E-04	7.50E-04	-3.75E-04	-3.75E-04
2.50E+00	-1.00E+02	2.50E-04	-2.25E-04	-2.25E-04	7.50E-04	-3.75E-04	-3.75E-04
2.75E+00	-1.50E+02	-7.50E-04	2.25E-04	2.25E-04	-7.14E-13	7.02E-13	1.17E-14
3.00E+00	-2.00E+02	-1.75E-03	6.75E-04	6.75E-04	-7.50E-04	3.75E-04	3.75E-04
3.25E+00	-1.50E+02	-1.50E-03	6.00E-04	6.00E-04	-7.50E-04	3.75E-04	3.75E-04
3.50E+00	-1.00E+02	-1.25E-03	5.25E-04	5.25E-04	-7.50E-04	3.75E-04	3.75E-04
3.75E+00	-5.00E+01	-1.00E-03	4.50E-04	4.50E-04	-7.50E-04	3.75E-04	3.75E-04
4.00E+00	3.26E-09	-7.50E-04	3.75E-04	3.75E-04	-7.50E-04	3.75E-04	3.75E-04
4.25E+00	5.00E+01	-5.00E-04	3.00E-04	3.00E-04	-7.50E-04	3.75E-04	3.75E-04
4.50E+00	1.00E+02	-2.50E-04	2.25E-04	2.25E-04	-7.50E-04	3.75E-04	3.75E-04
4.75E+00	1.50E+02	7.50E-04	-2.25E-04	-2.25E-04	6.51E-12	-2.64E-12	-3.87E-12
5.00E+00	2.00E+02	1.75E-03	-6.75E-04	-6.75E-04	7.50E-04	-3.75E-04	-3.75E-04

Let us define certain sizes first of all:

- the diverter of the tensor of the constraints: $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3} tr(\boldsymbol{\sigma}).\boldsymbol{I}$ where I is the matrix identity
- the diverter of the tensor of the deformations: $\boldsymbol{e} = \boldsymbol{\epsilon} - \frac{1}{3} tr(\boldsymbol{\epsilon}).\boldsymbol{I}$ where I is the matrix identity

It is stressed that the results in interval of time between 0 and 1 second are for the monotonous part of the loading and are not taken into account in the calculation of the sizes for the cyclic behavior.

Then, here reference solutions for:

`'DEPSPE'`: half-amplitude of the equivalent plastic deformation:

$$\frac{\Delta\varepsilon_{eq}^{p}}{2}=\frac{1}{2}max_{t1}max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2)):(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2))}=7.5\text{E}-4$$

`'EPSPR1'`: half-amplitude of the first principal deformation (with the taking into account of the sign):

$$\frac{\epsilon_{max}^{1}-\epsilon_{min}^{1}}{2}=7.625\text{E}-04$$

`'SIGNM1'` : maximum normal constraint as regards the principal deformation:

$$max_t(\boldsymbol{\sigma}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))=200\,MPa$$

`'APHYDR'` : half amplitude of the hydrostatic pressure (P_a)

$$P_a=\frac{P_{max}-P_{min}}{2}=66.6666\,MPa$$

`'DENDIS'`: density of dissipated energy:

$$W_{cy}=\int_{cylce}\boldsymbol{\sigma}:\dot{\boldsymbol{\epsilon}}^{p}\,dt=0.45$$

`'DENDIE'`: density of energy of the elastic distortions:

$$W_e=\int_{cylce}\langle\boldsymbol{s}:\dot{\boldsymbol{e}}^{e}\langle\,dt=0.173333$$

`'DSIGEQ'`: half-amplitude of the equivalent constraint:

$$\frac{\Delta\sigma_{eq}}{2}=\frac{1}{2}max_{t1}max_{t2}\sqrt{\frac{3}{2}(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2)):(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2))}=200\,MPa$$

`'SIGPR1'`: half-amplitude of the first principal constraint (with the taking into account of the sign):

$$\frac{\sigma_{max}^{1}-\sigma_{min}^{1}}{2}=100\,MPa$$

`'EPSNM1'`: maximum normal deformation as regards the principal constraint:

$$max_t(\boldsymbol{\epsilon}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))=1.75\text{E}-3$$

`'INVA2S'`: half-amplitude of the second invariant of the deformation:

$$J_2(\Delta\epsilon)=\frac{1}{2}max_{t1}max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2)):(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2))}=1.616666\text{E}-3$$

`'DSITRE'`: half-amplitude of the Tresca half-constraint:

$$\frac{\sigma_{max}^{Tresca}-\sigma_{min}^{Tresca}}{4}=50\,MPa$$

`'DEPTRE'`: half-amplitude of the Tresca half-deformation:

$$\frac{\epsilon_{max}^{Tresca}-\epsilon_{min}^{Tresca}}{4}=6.0625\text{E}-4$$

`'EPSPAC'`: plastic deformation accumulated:

$$p=3.67423\text{E}-3$$

`'DEPSEE'`: half-amplitude of the equivalent elastic strain:

$$\frac{\Delta\varepsilon_{eq}^{p}}{2}=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{2}{3}\left(\boldsymbol{\epsilon}^{e}(t_1)-\boldsymbol{\epsilon}^{e}(t_2)\right):\left(\boldsymbol{\epsilon}^{e}(t_1)-\boldsymbol{\epsilon}^{e}(t_2)\right)}=0.00086666666$$

2.6 Results of reference for modeling E

The results are got with the node $N1$.

t	σ_{xx}	σ_{yy}	ϵ_{xx}^{tot}	ϵ_{yy}^{tot}	ϵ_{zz}^{tot}	ϵ_{xx}^{p}	ϵ_{yy}^{p}	ϵ_{zz}^{p}
0.00E+0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.50E-1	3.00E+02	-2.00E+02	1.80E-03	-1.45E-03	-1.50E-04	-6.73E-19	-2.69E-18	2.84E-19
5.00E-1	6.00E+02	-4.00E+02	3.60E-03	-2.90E-03	-3.00E-04	-4.71E-19	-4.39E-18	2.61E-19
7.50E-1	9.00E+02	-6.00E+02	1.10E-02	-9.26E-03	-1.15E-03	5.61E-03	-4.91E-03	-7.01E-04
1.00E+0	1.20E+03	-8.00E+02	1.88E-02	-1.60E-02	-2.05E-03	1.16E-02	-1.02E-02	-1.45E-03
1.25E+0	9.00E+02	-6.00E+02	1.70E-02	-1.45E-02	-1.90E-03	1 .1 6 E-0 2	- 1. 02E-02	- 1. 4 5th-03
1.50E+0	6.00E+02	-4.00E+02	1.52E-02	-1.31E-02	-1.75E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
1.75E+0	3.00E+02	-2.00E+02	1.34E-02	-1.16E-02	-1.60E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
2.00E+0	2.30E-13	-3.07E-13	1.16E-02	-1.02E-02	-1.45E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
2.25E+0	3.00E+02	-2.00E+02	1.34E-02	-1.16E-02	-1.60E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
2.50E+0	6.00E+02	-4.00E+02	1.52E-02	-1.31E-02	-1.75E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
2.75E+0	9.00E+02	-6.00E+02	1.70E-02	-1.45E-02	-1.90E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
3.00E+0	6.00E+02	-4.00E+02	1.52E-02	-1.31E-02	-1.75E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
3.25E+0	3.00E+02	-2.00E+02	1.34E-02	-1.16E-02	-1.60E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03
3.50E+0	-4.35E-14	-8.66E-13	1.16E-02	-1.02E-02	-1.45E-03	1. 1 6th-02	-1.02E-02	- 1. 4 5th-03

It is stressed that the results in interval of time between 0 and 1 second are for the monotonous part of the loading and are not taken into account in the calculation of the sizes for the cyclic behavior.

For the reference solutions we calculated the damage (via `ENDO1`) with the various sizes available for a nonperiodic loading:

`'SIPR1_1'`: first principal constraint of the first top of under cycle ($\sigma_1(1)$) and
`'SIPR1_2'`: first principal constraint of the second top of under cycle ($\sigma_1(2)$):

One uses the half-amplitude of the constraint of the top of the first cycle, namely $600\,MPa$ and the half-amplitude of the constraint of the top of the second cycle, namely $450\,Mpa$ then one adds the damages associated with these two amplitudes.

One evaluates the criterion here following: $\frac{|SIPR1-SIPR2|}{2}$

- with the formula of Basquin: $\text{grandeur équivalente}=4098.3\times(NBRUP^{-0.2693})$
  one finds $NBRUP\,1=1255$ and for the damage: $D1=7.968963\text{E}-04$ then $NBRUP\,2=3652$ and for the damage: $D2=2.738186\,E-04$ that is to say a total damage equal to: $D=1.0707149\text{E}-03$

- with an interpolation of the curve of Wöhler:
  one finds $NBRUP1=742$ and for the damage: $D=1.347073\text{E}-03$. One finds $NBRUP2=1742$ and for the damage: $D=5.741842\text{E}-04$ that is to say a total damage equal to: $D=1.9212575\text{E}-03$

`'SITN1_1'`: normal constraint on the level associated with $\epsilon_1^{tot}(1)$ first top of the under-cycle and
`'SITN1_2'`: normal constraint on the level associated with $\epsilon_1^{tot}(2)$ second top of the under-cycle:

One uses the half-amplitude of the associated constraint $\epsilon_1^{tot}(1)$, namely $600\,MPa$ and the half-amplitude of the associated constraint $\epsilon_1^{tot}(2)$, namely $450\,MPa$ then one adds the damages associated with these two amplitudes.
One evaluates the criterion here following: $\frac{|SITN1-SITN2|}{2}$

- with the formula of Basquin: $\text{grandeur équivalente}=4098.3\times(NBRUP^{-0.2693})$
  one finds $NBRUP\,1=1255$ and for the damage: $D1=7.968963\text{E}-04$ then $NBRUP2=3652$ and for the damage: $D2=2.738186\,E-04$ that is to say a total damage equal to: $D=1.0707149\text{E}-03$

- with an interpolation of the curve of Wöhler:
  one finds $NBRUP1=742$ and for the damage: $D=1.347073\text{E}-03$ one finds $NBRUP2=1742$ and for the damage: $D=5.741842\text{E}-04$ that is to say a total damage equal to: $D=1.9212575\text{E}-03$

`'SIPN1_1'`: normal constraint on the level associated with $\epsilon_1^p(1)$ first top of the under-cycle and
`'SIPN1_2'`: normal constraint on the level associated with $\epsilon_1^p(2)$ second top of the under-cycle:

One uses the value the half-amplitude of the associated constraint $\epsilon_1^p(1)$, namely $600\,Mp$ has and the half-amplitude of the associated constraint $\epsilon_1^p(2)$, namely $450\,Mpa$
One evaluates the criterion here following: $\frac{SIPN1-SIPN2}{2}$

- with the formula of Basquin: $\text{grandeur\_equivalente}=4098.3\times(NBRUP^{-0.2693})$
  one finds $NBRUP1=1255$ and for the damage: $D1=7.968963\text{E}-04$ then $NBRUP\,2=3652$ and for the damage: $D2=2.738186\,E-04$ that is to say a total damage equal to: $D=1.0707149\text{E}-03$

- with an interpolation of the curve of Wöhler:
  one finds $NBRUP1=742$ and for the damage: $D=1.347073\text{E}-03$ one finds $NBRUP2=1742$ and for the damage: $D=5.741842\text{E}-04$ that is to say a total damage equal to: $D=1.9212575\text{E}-03$

`'SIGEQ_1'`: equivalent constraint of the first top of the under-cycle $\sigma_{eq}(1)$ and
`' SIGEQ_2 '`: equivalent constraint of the second top of the under-cycle $\sigma_{eq}(2)$:

One calculates the equivalent constraint for SIGEQ_1:

$$\frac{\Delta\sigma_{eq}}{2}=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{3}{2}(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2)):(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2))}=871.78\,MPa$$

then for `SIGEQ_2` :

$$\frac{\Delta\sigma_{eq}}{2}=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{3}{2}(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2)):(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2))}=653.83\,MPa$$

And one evaluates the criterion here following: $\frac{SIGEQ1-SIGEQ2}{2}$

- with the formula of Basquin: $\text{grandeur\_equivalente}=4098.3\times(NBRUP^{-0.2693})$
  one finds $NBRUP1=31$ 3 and for the damage: $D1=3.1908789\,E-03$ then $NBRUP2=912$ and for the damage: $D2=1.0964061\,E-03$ that is to say a total damage equal to: $D=4.287285\,E-03$

- with an interpolation of the curve of Wöhler:
  one finds $NBRUP1=244$ and for the damage: $D=4.1\text{E}-03$ one finds $NBRUP2=582$ and for the damage: $D=1.7175686\text{E}-03$ that is to say a total damage equal to: $D=5.8176\,E-03$

2.7 Results of reference for modeling F

The results are got with the node $N1$.

- **Results obtained with `SOL_NL` :**

t	σ_{xx}	σ_{yy}	ϵ_{xx}^{tot}	ϵ_{yy}^{tot}	ϵ_{zz}^{tot}
0.00E+0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
2.50E-1	3.00E+02	-1.50E+02	1.725E-03	-1.20E-03	-2.25E-04
5.00E-1	6.00E+02	-3.00E+02	3.45E-03	-2.40E-03	-4.50E-04
7.50E-1	9.00E+02	-4.50E+02	5.175E-03	-3.60E-03	-6.75E-04
1.00E+0	1.20E+03	-6.00E+02	6.90E-03	-4.80E-03	-9.00E-04
1.25E+0	9.00E+02	-4.50E+02	5.175E-03	-3.60E-03	-6.75E-04
1.50E+0	6.00E+02	-3.00E+02	3.45E-03	-2.40E-03	-4.50E-04
1.75E+0	3.00E+02	-1.50E+02	1.725E-03	-1.20E-03	-2.25E-04
2.00E+0	8.82E-26	8.82E-26	-7,92E-33	- 1. 39 E 32	-1.85E-32
2.25E+0	3.00E+02	-1.50E+02	1.725E-03	-1.20E-03	-2.25E-04
2.50E+0	6.00E+02	-3.00E+02	3.45E-03	-2.40E-03	-4.50E-04
2.75E+0	9.00E+02	- 4.5 0E+ 02	5 .1 75 E-0 3	- 3. 60 E-0 3	-6.75E-04
3.00E+0	6.00E+02	-3.00E+02	3.45E-03	-2.40E-03	-4.50E-04
3.25E+0	3.00E+02	-1.50E+02	1.725E-03	-1.20E-03	-2.25E-04
3.50E+0	7.88E-14	-5.15E-13	1.26E-18	-2.60E-18	3.52E-19

'`EPSN1_1`**' :** normal deformation on the level associated with $\sigma_1(1)$ first top of the under-cycle and
' `EPSN 1_ 2` **' :** normal deformation on the level associated with $\sigma_1(2)$ second top of the under-cycle:

One uses the total half-deformation most important met at the time of the first under cycle, that is to say here 0.00345 and the total half-deformation most important met at the time of the second under cycle: 0.0025875 .

And one evaluates the criterion here following: $\frac{|EPSN1-EPSN2|}{2}$

- with the formula of Manson: $\text{grandeur\_equivalente}=0{,}022524751\,x\,(NBRUP^{-0{,}1619})$

one finds $NBRUP1=107892$ and for the damage: $D1=9.268535\,E-06$ one finds $NBRUP2=637811$ and for the damage: $D2=1.5678624479\,E-06$ that is to say a total damage equal to: $D=1.08363973\,E-05$

- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=3636$ 4 and for the damage: $D1=2.74994\,E-05$ one finds $NBRUP2=193932$ and for the damage: D2 = 5.156443564 E-06 is a total damage equal to: $D=3.2655868578\,E-05$

'ETPR1_1' : first principal total deflection of the first top of the under-cycle $\epsilon_1^{tot}(1)$ and
'ETPR1_2' : first principal total deflection of the second top of the under-cycle $\epsilon_1^{tot}(2)$:

One uses the total half-deformation most important met at the time of the first under cycle, that is to say here 0.00345 and the total half-deformation most important met at the time of the second under cycle: 0.0025875 .

And one evaluates the criterion here following: $\frac{|ETPR1-ETPR2|}{2}$

- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\text{x}(NBRUP^{-0.1619})$
  one finds $NBRUP1=107892$ and for the damage: $D1=9.268535\,E-06$ one finds $NBRUP2=637811$ and for the damage: $D2=1.5678624479\,E-06$ that is to say a total damage equal to: $D=1.08363973\,E-05$

- with an interpolation of the curve of Manson-Whetstone sheath:

one finds $NBRUP1=36364$ and for the damage: $D1=2.74994\,E-05$ one finds $NBRUP2=193932$ and for the damage: $D2=5.156443564\,E-06$ that is to say a total damage equal to: $D=3.2655868578\,E-05$

'ETEQ_1' : equivalent total deflection of the first top of the under-cycle $\epsilon_{eq}^{tot}(1)$ and
'ETEQ_2' : equivalent total deflection of the second top of the under-cycle $\epsilon_{eq}^{tot}(2)$

One calculates the equivalent total deflection for the first under-cycle with the formula: $\epsilon_{eq}=\sqrt{\frac{2}{3}e:e}$ and one finds $\frac{1}{2}\epsilon_{eq}^{tot}(1)=0.0034394767$. For the second under-cycle: $\frac{1}{2}\epsilon_{eq}^{tot}(2)=0.0025796$

- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\times(NBRUP^{-0.1619})$
  one finds $NBRUP1=109947$ and for the damage: $D1=9.09528647\,E-06$ one finds $NBRUP2=649960$ and for the damage: $D2=1.5385558\,E-06$ that is to say a total damage equal to: $D=1.0633842276096\,E-05$

- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=36929$ and for the damage: $D1=2.707933\,E-05$ one finds $NBRUP2=197585$ and for the damage: $D2=5.061111575\,E-06$ that is to say a total damage equal to: $D=3.214044324622\,E-05$

- **Results obtained with `SOL_NL2` :**

t	σ_{xx}	ϵ_{xx}^{tot}	ϵ_{yy}^{tot}	ϵ_{zz}^{tot}
0.00E+0	0.00E+00	0.00E+00	0.00E+00	0.00E+00
5.00E-1	6.00E+02	3.00E-03	-9.00E-04	-9.00E-04
1.00E+0	1.20E+03	1.05E-02	-4.05E-03	-4.05E-03
1.50E+0	6.00E+02	7.50E-03	-3.15E-03	-3.15E-03
2.00E+0	-9 . 12th-1 3	4.50E-03	-2.25E-03	-2.25E-03
2.5625E+0	-6.75E+02	-1.91E-13	-6.75E-04	-6.75E-04
3.0625E+0	1.125E+02	3.9375E-03	-1.86E-03	-1.86E-03
3.5625E+0	9.00E+02	7.875E-03	-3.0375E-03	-3.0375E-03
4.0625E+0	1.125E+02	3.9375E-03	- 1.86 E-03	- 1.86 E-03
4.5625E+0	-6.75E+02	-1.91E-13	- 6.75 E-0 4	-6.75E-04

`'EPSN1_1'` : normal deformation on the level associated with $\sigma_1(1)$ first top of the under-cycle and
`'EPSN1_2'` : normal deformation on the level associated with $\sigma_1(2)$ second top of the under-cycle:

The dem is usedthe total l-deformation most important met at the time of the first under cycle, is here 0.00525 and the half total deflection most important met at the time of the second under cycle: 0.0039375 .

And one evaluates the criterion here following: $\frac{|EPSN1-EPSN2|}{2}$

- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\times(NBRUP^{-0.1619})$
  one finds $NBRUP1=8067$ and for the damage: $D1=1.239547\,E-04$ one finds $NBRUP2=47691$ and for the damage: $D2=2.0968145\,E-05$ that is to say a total damage equal to: $D=1.449229\,E-04$
- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=5260$ and for the damage: $D1=1.901065\,E-04$ one finds $NBRUP2=19698$ and for the damage: $D2=5.0767\,E-05$ that is to say a total damage equal to: $D=2.408735\,E-04$

`'ETPR1_1'` : first principal total deflection of the first top of the under-cycle $\epsilon_1^{tot}(1)$ and
`'ETPR1_2 '`: first principal total deflection of the second top of the under-cycle $\epsilon_1^{tot}(2)$:

One uses the total half-deformation most important met at the time of the first under cycle, that is to say here 0.00345 and the total half-deformation most important met at the time of the second under cycle: 0.0025875 .

And one evaluates the criterion here following: $\frac{|ETPR1-ETPR2|}{2}$

- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\text{x}(NBRUP^{-0.1619})$
  one finds $NBRUP1=8067$ and for the damage: $D1=1.239547\,E-04$ one finds $NBRUP2=47691$ and for the damage: $D2=2.0968145\,E-05$ that is to say a total damage equal to: $D=1.449229\,E-04$
- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=5260$ and for the damage: $D1=1.901065\,E-04$ one finds $NBRUP2=19698$ and for the damage: $D2=5.0767\,E-05$ that is to say a total damage equal to: $D=2.408735\,E-04$

`'ETEQ_1'` : equivalent total deflection of the first top of the under-cycle $\epsilon_{eq}^{tot}(1)$ and

`'ETEQ_2'` : equivalent total deflection of the second top of the under-cycle $\epsilon_{eq}^{tot}(1)$

One calculates the equivalent total deflection for the first under-cycle with the formula: $\epsilon_{eq}=\sqrt{\frac{2}{3}e:e}$ and one finds $\frac{1}{2}\epsilon_{eq}^{tot}(1)=0.004625$. For the second under-cycle, one finds $\frac{1}{2}\epsilon_{eq}^{tot}(2)=0.0034125$

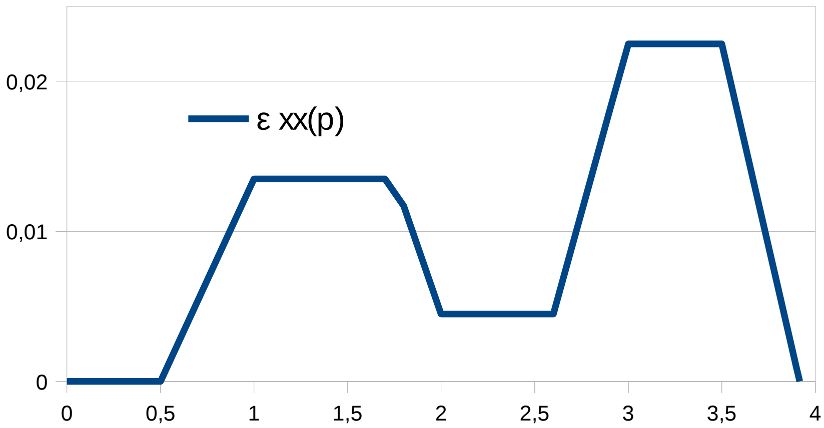
- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\times(NBRUP^{-0.1619})$
  one finds $NBRUP1=17650$ and for the damage: $D1=5.6657\,E-05$ one finds $NBRUP2=115427$ and for the damage: $D2=8.66351\,E-06$ that is to say a total damage equal to: $D=6.53204991\,E-05$

- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=9411$ and for the damage: $D1=1.062557\,E-04$ one finds $NBRUP2=38423$ and for the damage: $D2=2.60258665\,E-05$ that is to say a total damage equal to: $D=1.322816\,E-04$

- **Results obtained with `SOL_NL3` :**

t	σ_{xx}	ϵ_{xx}^{tot}	ϵ_{yy}^{tot}	ϵ_{zz}^{tot}	ϵ_{xx}^{p}	ϵ_{yy}^{p}	ϵ_{zz}^{p}
0.00E+0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
5.00E-1	9.00E+02	4.50E-03	-1.35E-03	-1.35E-03	1.51E-18	1.09E-18	-7.51E-19
1.00E+0	1.80E+03	2.25E-02	-9.45E-03	-9.45E-03	1.35E-02	-6.75E-03	-6.75E-03
1.50E+0	6.00E+02	1.65E-02	-7.65E-03	-7.65E-03	1.35E-02	-6.75E-03	-6.75E-03
2.00E+0	-6.00E+02	1.50E-03	-1.35E-03	-1.35E-03	4.50E-03	- 2. 25 E-0	- 2. 25 E-0
2.50E+0	9.00E+02	9.00E-03	-3.60E-03	-3.60E-03	4 . 50 E-0 3	3	3
3.00E+0	2.40E+03	3.45E-02	-1.485E-02	-1.485E-02	2 . 25 E-02	-2.25E-03	-2.25E-03
3.50E+0	6.00E+02	2.55E-02	-1.215E-02	-1.215E-02	2 . 25 E-02	-1.125E-02	-1.125E-02
3.91667E+0	-9.00E+02	-4.50E-03	1.35 E-03	1.35E-03	-5.55 E-11	-1.125E-02	-1.125E-02
						2.69E-11	2 . 86 E 11

With this intention a better idea of what interests us here, namely ϵ_{xx}^{p} since the maximum values of the plastic deformation are according to the direction xx , here evolution of ϵ_{xx}^{p} according to time:



`'EPPR1_1'` : first principal plastic deformation of the first top of the under-cycle $\epsilon_1^{p}(1)$ and

'EPPR1_2': first principal plastic deformation of the second top of the under-cycle $\epsilon_1^p(2)$:

One evaluates the criterion here following: $\frac{|EPPR1-EPPR2|}{2}$

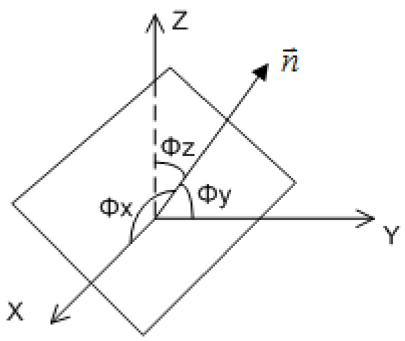
Attention, here the value of ϵ_{xx}^p is not worthless at the end of the first under cycle, therefore for $\frac{EPPR1}{2}$ one obtains: $\frac{0.0135-0.0045}{2}=0.0045$. For $\frac{EPPR2}{2}$, one obtains more directly: $0{,}01125$.

- with the formula of Manson: $\text{grandeur\_equivalente}=0.022524751\times(NBRUP^{-0.1619})$
  one finds $NBRUP1=20905$ and for the damage: $D1=4.7836\,E-05$ one finds $NBRUP2=73$ and for the damage: $D2=1.37307\,E-02$ that is to say a total damage equal to: $D=1.377855\,E-02$

- with an interpolation of the curve of Manson-Whetstone sheath:
  one finds $NBRUP1=10672$ and for the damage: $D1=9.37\,E-05$ one finds $NBRUP2=478$ and for the damage: $D2=2.092\,1444\,E-03$ that is to say a total damage equal to: $D=2.185844\,E-03$

2.8 Results of reference for modeling G

The analytical solutions of the orientation of the plan criticizes are in [bib4]. Orientation of the plan critical is defined by angles (ϕ_x , ϕ_y , ϕ_z) between the normal vector of the critical plan and axes as shown in the figure above.



One notes that for the criteria DANG_VAN _MODI_AC and of MATAKE_MODI_AC , the critical plan is it plan maximum shearing. For the criteria of DANG_VAN_MODI_AV , of MATAKE_MODI_AC and of FATESOCI_MODI_AV the critical plan is the plan of the maximum damage.

With the conditions of the loading listed in Section 1.4.6, the analytical solutions of reference for this modeling are the following ones

- Criterion of DANG_VAN _MODI_AC
  The angle enters the vector normal of the critical plan and the axis Z is 45 degrees.
- Criterion of MATAKE_MODI_AC
  The angle enters the vector normal of the critical plan and the axis Z is 45 degrees.
- Criterion of DANG_VAN_MODI_AV
  The angle enters the vector normal of the critical plan and the axis Z is 45 degrees.
- Criterion of MATAKE_MODI_AV
  The angle enters the vector normal of the critical plan and the axis Z according to α is

$$\phi_z=\arccos(\frac{\sqrt{2}}{2}\sqrt{1-\sqrt{1-\frac{1}{a^2+2a^2\alpha+a^2\alpha^2+1}}})$$

where a is a property matérial obtained by the parameter MATAKE_A and $a=0.05$ is used in this case test.

- Criterion of FATESOCI_MODI_AV
  The angle enters the vector normal of the critical plan and the axis Z according to α is

$$\phi_z = \arccos\left(\frac{\sqrt{2}}{4}\sqrt{5+A_1-\sqrt{1+A_1^2+8}}\right)$$

with $A_1=\frac{2S_y}{a\sigma_{x,a}(1+\alpha)}$. Elastic limit $S_y=208\,MPa$ and $a=0.05$. Therefore, the parameter of FATSOC_A is $a/S_y=0.00024$

2.9 Results of reference for modeling H

In this modeling, the results of reference analytical and are given in the table below.

Let us define certain sizes first of all:

- the diverter of the tensor of the constraints: $\boldsymbol{s}=\boldsymbol{\sigma}-\frac{1}{3}tr(\boldsymbol{\sigma}).\boldsymbol{I}$ where I is the matrix identity
- the diverter of the tensor of the deformations: $\boldsymbol{e}=\boldsymbol{\epsilon}-\frac{1}{3}tr(\boldsymbol{\epsilon}).\boldsymbol{I}$ where I is the matrix identity

It is stressed that the results in interval of time between 0 and 1 second are for the monotonous part of loading and are not taken into account in the calculation of the sizes for the cyclic behavior.

It is noted that there exist two types of shearing strains: the type of engineering γ_{ij} ($i\neq j$) and it tensorial type ϵ_{ij} ($i\neq j$). To note that $\gamma_{ij}=2\epsilon_{ij}$. For 'DGAMCR', 'MGAMCR', 'MGAMPC', one used the deformations of the type of engineering γ_{ij} .

Sizes	'SIGEQ1'	'ENDO1'	'NBRUP1'	'VNM1X', 'VNM1Y', 'VNM1Z'
'DTAUCR'	100 MPa	1.028E-6	9.73E5	(0,707, -0,707), 0,707.0
'DGAMCR'	9.73E5	1.583E-4	6.3163E3	(0,707, -0,707), 0,707.0
'DSINCR'	200	1.348E-5	7.418E4	(- 1.1), 0.0174,0
'DEPNCR'	1.75E-3	2.11E-5	4.74E4	(- 1, 1), 0.0174,0
'MTAUCR'	100 MPa	1.028E-6	9.73E5	(0,707, -0,707), 0,707.0
'MGAMCR'	9.73E5	1.583E-4	6.3163E3	(0,707, -0,707), 0,707.0
'MSINCR'	200	1.348E-5	7.418E4	(- 1.1), 0.0174,0
'MEPNCR'	1.75E-3	2.11E-5	4.74E4	(- 1, 1), 0.0174,0
'DGAMPC'	1.125E-3	1.3782E-6	7.256E5	(0,707, -0,707), 0,707.0
'DEPNPC'	0.75E-3	1.126E-7	8.88E6	(- 1.1), 0.0174,0
'MGAMPC'	1.125E-3	1.3782E-6	7.256E5	(0,707, -0,707), 0,707.0
'MEPNPC'	0.75E-3	1.126E-7	8.88E6	(- 1.1), 0.0174,0

2.10 Results of reference for modeling I

See results of modeling F (2.7).

2.11 Bibliographical references

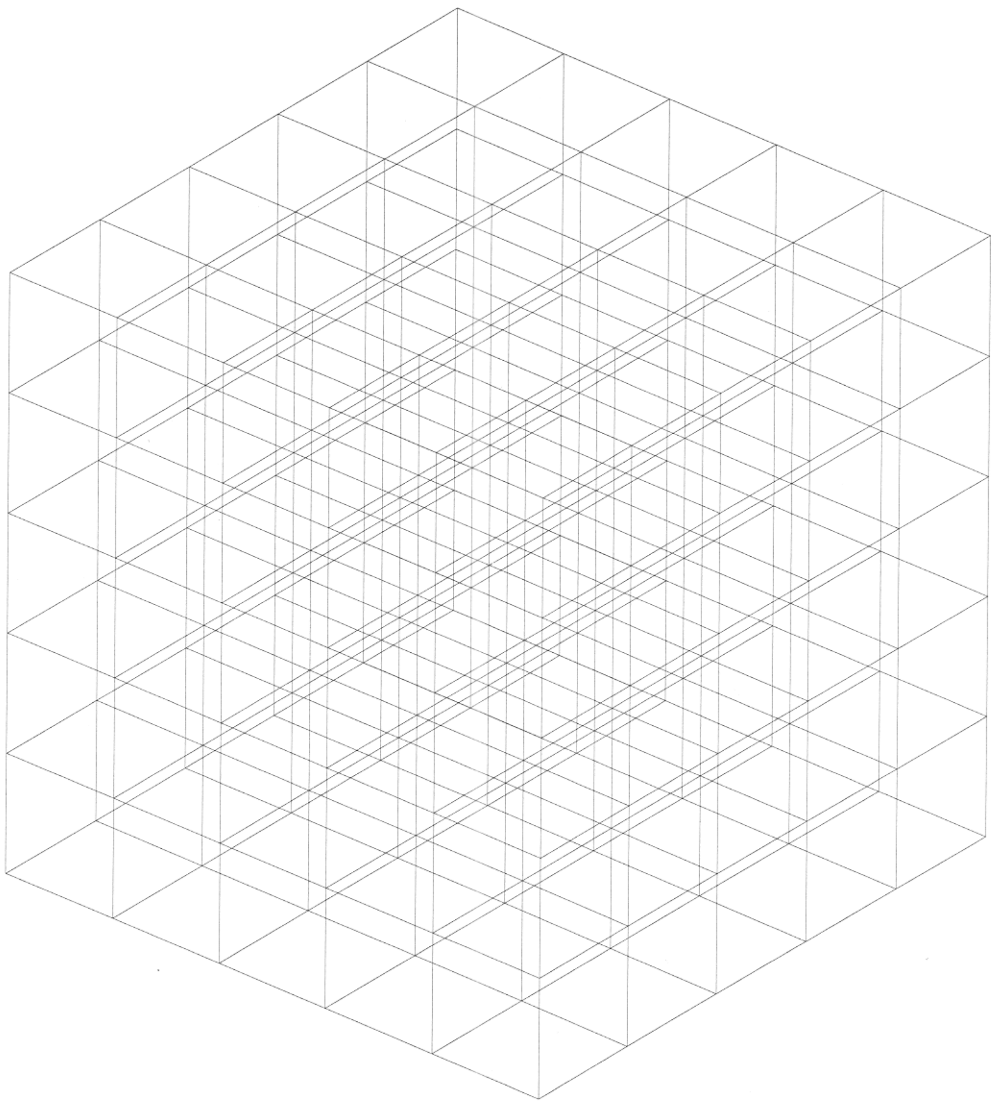
1. ANGLES J. “multiaxial Criteria of starting in fatigue to great number D cycles, critical plan, DANG VAN”, Project FATMAV, Notes EDF HT-64/03/015A, 2003
2. ANGLES J., PAPACONSTANTINOU T. “multiaxial Criteria of starting in fatigue to great number D cycles under nonperiodic loading”, Project FATMAV, Note EDF HT-64/04/006A, 2004
3. ANGLES J. “Synthesis on the criteria of starting in fatigue multiaxial to great number D cycles developed in *Code_Aster* ”, Project Tires Thermal, Notes EDF HT-64/05/019A, 2005
4. LEI B., TRAN V. - X. “Carryforward one the 2Nd semi-anual meeting for the collaborative PhD project between EDF R & D and Tsinghua University (July 2012 – January 2013) “, Report CR-AMA-13.021

3 Modeling A

3.1 Characteristics of modeling

This CAS-test tests criteria MATAKE_MODI_AC, DANG_VAN_MODI_AC, VMIS_TRESCA, criterion formulates some for periodic and biaxial loading proportional;

3.2 Characteristics of the grid



Modeling 3D : 125 quadratic elements of volume: HEXA8.

Figure of the grid of the cube

The grid of the cube was obtained starting from the version 2000 of maillor GIBI.

Many nodes: 216
Many meshs: 465

3.3 Sizes tested and results

- **Criterion `'MATAKE_MODI_AC'` and associated criterion in formula:**

For the results with the node $N1$ and with the mesh $M60$ (not Gauss 3)

Test of the constraints to resulting from `STAT_NON_LINE` :

Identification	Type of reference	Value of reference	Tolerance
σ_{xx} **with the node** $N1$	'ANALYTICAL'	-100	1.0E-10
σ_{yy} **with the node** $N1$	'ANALYTICAL'	200	1.0E-10
σ_{xx} **with the mesh** $M60$ **(not Gauss 3)**	'ANALYTICAL'	-100	1.0E-10
σ_{yy} **with the mesh** $M60$ **(not Gauss 3)**	'ANALYTICAL'	200	1.0E-10

The option COURBE_GRD_VIE = 'WOHLER' and
The option COURBE_GRD_VIE = 'FORMES_VIE' and FORMULE_VIE = WHOL:

Identification	Type of reference	Value of reference	Tolerance
$\Delta\tau(n_1)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_1	'ANALYTICAL'	-7.071068E – 01	1.0E-10
component y of n_1	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_1	'ANALYTICAL'	0.0	1.0E-10
$N_{\max}(n_1)$	'ANALYTICAL'	5.000000E+01	1.0E-10
$N_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\varepsilon_{\max}(n_1)$	'ANALYTICAL'	1.750000E – 04	1.0E-10
$\varepsilon_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\sigma_{eq}(n_1)$	'ANALYTICAL'	3.000000E+02	1.0E-10
$Nb_{cr}(n_1)$	'ANALYTICAL'	1.094600E+04	1.0E-10
$ENDO(n_1)$	'ANALYTICAL'	9.135647E- 05	1.0E-10
$\Delta\tau(n_2)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_2	'ANALYTICAL'	7.071068E – 01	1.0E-10
component y of n_2	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_2	'ANALYTICAL'	0.0	1.0E-10
$N_{\max}(n_2)$	'ANALYTICAL'	5.000000E+01	1.0E-10

$N_m(n_2)$	`'ANALYTICAL'`	0.0	1.0E-10
$\varepsilon_{max}(n_2)$	`'ANALYTICAL'`	1.750000E – 04	1.0E-10
$\varepsilon_m(n_2)$	`'ANALYTICAL'`	0.0	1.0E-10
$\sigma_{eq}(n_2)$	`'ANALYTICAL'`	3.000000E+02	1.0E-10
$Nb_{cr}(n_2)$	`'ANALYTICAL'`	1.094600E+04	1.0E-10
$ENDO(n_2)$	`'ANALYTICAL'`	9.135647E – 05	1.0E-10

The option `COURBE_GRD_VIE = 'FORMES_VIE'` and `FORMULE_VIE = WHOL_F`:

Identification	Type of reference	Value of reference	Tolerance
$\Delta\tau(n_1)$	`'ANALYTICAL'`	1 . 500000E + 02	1.0E-10
component x of n_1	`'ANALYTICAL'`	– 7.071068E – 01	1.0E-10
component y of n_1	`'ANALYTICAL'`	7.071068E – 01	1.0E-10
component z of n_1	`'ANALYTICAL'`	0.0	1.0E-10
$N_{max}(n_1)$	`'ANALYTICAL'`	5.000000E+01	1.0E-10
$N_m(n_1)$	`'ANALYTICAL'`	0.0	1.0E-10
$\varepsilon_{max}(n_1)$	`'ANALYTICAL'`	1.750000E – 04	1.0E-10
$\varepsilon_m(n_1)$	`'ANALYTICAL'`	0.0	1.0E-10
$\sigma_{eq}(n_1)$	`'ANALYTICAL'`	3.000000E+02	1.0E-10
$Nb_{cr}(n_1)$	`'ANALYTICAL'`	1.094600E+04	4.0E-03
$ENDO(n_1)$	`'ANALYTICAL'`	1.6519E+04	4.0E-03
$\Delta\tau(n_2)$	`'ANALYTICAL'`	6.05356E-05	1.0E-10
component x of n_2	`'ANALYTICAL'`	7.071068E – 01	1.0E-10
component y of n_2	`'ANALYTICAL'`	7.071068E – 01	1.0E-10
component z of n_2	`'ANALYTICAL'`	0.0	1.0E-10
$N_{max}(n_2)$	`'ANALYTICAL'`	5.000000E+01	1.0E-10
$N_m(n_2)$	`'ANALYTICAL'`	0.0	1.0E-10
$\varepsilon_{max}(n_2)$	`'ANALYTICAL'`	1.750000E – 04	1.0E-10
$\varepsilon_m(n_2)$	`'ANALYTICAL'`	0.0	1.0E-10

$\sigma_{eq}(n_2)$	'ANALYTICAL'	3.000000E+02	1.0E-10
$Nb_{cr}(n_2)$	'ANALYTICAL'	1.6519E+04	4.0E-03
$ENDO(n_2)$	'ANALYTICAL'	6.05356E-05	4.0E-03

- **Criterion `'DANG_VAN_MODI_AC'` and associated criterion in formula:**

For the results **with the node** $N1$ and with **mesh** $M60$ **(not Gauss 3)**

The option `COURBE_GRD_VIE = 'WOHLER'` and

The option `COURBE_GRD_VIE = 'FORMES_VIE'` and `FORMULE_VIE = 'WHOL'` :

Identification	Type of reference	Value of reference	Tolerance
$\Delta\tau(n_1)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_1	'ANALYTICAL'	7.071068E – 01	1.0E-10
component y of n_1	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_1	'ANALYTICAL'	0.0	1.0E-10
$N_{max}(n_1)$	'ANALYTICAL'	5.000000E+01	1.0E-10
$N_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\varepsilon_{max}(n_1)$	'ANALYTICAL'	1.750000E – 04	1.0E-10
$\varepsilon_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\sigma_{eq}(n_1)$	'ANALYTICAL'	2.750000E+02	1.0E-10
$Nb_{cr}(n_1)$	'ANALYTICAL'	1.490300E+04	1.0E-10
$ENDO(n_1)$	'ANALYTICAL'	6.709959E – 05	1.0E-10
$\Delta\tau(n_2)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_2	'ANALYTICAL'	– 7.071068E – 01	1.0E-10
component y of n_2	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_2	'ANALYTICAL'	0.0	1.0E-10
$N_{max}(n_2)$	'ANALYTICAL'	5.000000E+01	1.0E-10
$N_m(n_2)$	'ANALYTICAL'	0.0	1.0E-10
$\varepsilon_{max}(n_2)$	'ANALYTICAL'	1.750000E – 04	1.0E-10
$\varepsilon_m(n_2)$	'ANALYTICAL'	0.0	1.0E-10
$\sigma_{eq}(n_2)$	'ANALYTICAL'	2.750000E+02	1.0E-10
$Nb_{cr}(n_2)$	'ANALYTICAL'	1.490300E+04	1.0E-10

$ENDO(n_2)$	'ANALYTICAL'	6.709959E – 05	1.0E-10

The option COURBE_GRD_VIE = 'FORMES_VIE' and FORMULE_VIE = WHOL_F:

Identification	Type of reference	Value of reference	Tolerance
$\Delta\tau(n_1)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_1	'ANALYTICAL'	7.071068E – 01	1.0E-10
component y of n_1	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_1	'ANALYTICAL'	0.0	1.0E-10
$N_{\max}(n_1)$	'ANALYTICAL'	5.000000E+01	1.0E-10
$N_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\varepsilon_{\max}(n_1)$	'ANALYTICAL'	1.750000E – 04	1.0E-10
$\varepsilon_m(n_1)$	'ANALYTICAL'	0.0	1.0E-10
$\sigma_{eq}(n_1)$	'ANALYTICAL'	2.750000E+02	1.0E-10
$Nb_{cr}(n_1)$	'ANALYTICAL'	2.2822E+04	5.0E-03
$ENDO(n_1)$	'ANALYTICAL'	4.381737E-05	5.0E-03
$\Delta\tau(n_2)$	'ANALYTICAL'	1 . 500000E + 02	1.0E-10
component x of n_2	'ANALYTICAL'	– 7.071068E – 01	1.0E-10
component y of n_2	'ANALYTICAL'	7.071068E – 01	1.0E-10
component z of n_2	'ANALYTICAL'	0.0	1.0E-10
$N_{\max}(n_2)$	'ANALYTICAL'	5.000000E+01	1.0E-10
$N_m(n_2)$	'ANALYTICAL'	0.0	1.0E-10
$\varepsilon_{\max}(n_2)$	'ANALYTICAL'	1.750000E – 04	1.0E-10
$\varepsilon_m(n_2)$	'ANALYTICAL'	0.0	1.0E-10
$\sigma_{eq}(n_2)$	'ANALYTICAL'	2.750000E+02	1.0E-10
$Nb_{cr}(n_2)$	'ANALYTICAL'	2.2822E+04	5.0E-03
$ENDO(n_2)$	'ANALYTICAL'	4.381737E-05	5.0E-03

- **Criterion ' VMIS_TRESCA '**

For nodes: N1; N206; Mesh: M60 (Not Gauss: 3)

Identification	Type of reference	Value of reference	Tolerance
σ_{xx} (Moment: 3)	'ANALYTICAL'	– 1.00000E+02	1.0E-10

σ_{yy} (Moment: 3)	`'ANALYTICAL'`	2.00000E+02	1.0E-10
σ_s (VMIS)	`'ANALYTICAL'`	529.15026E+02	1.0E-10
σ_s (TRESCA)	`'ANALYTICAL'`	600.00000E+02	1.0E-10

For nodes: N1; N206; Mesh: M60 (Not of Gauss: 7)

Identification	Type of reference	Value of reference	Tolerance
σ_{xx} (Moment: 3)	`'ANALYTICAL'`	– 1.00000E+02	1.0E-10
σ_{yy} (Moment: 3)	`'ANALYTICAL'`	2.00000E+02	1.0E-10
σ_s (VMIS)	`'ANALYTICAL'`	529.15026E+02	1.0E-10
σ_s (TRESCA)	`'ANALYTICAL'`	600.00000E+02	1.0E-10

- **Criterion formulates some finding `' VMIS_TRESCA '`**

Results **with the node** $N1$ and with **mesh** $M60$ **(not Gauss 3 and 7)**

Identification	Type of reference	Value of reference	Tolerance
σ_s (VMIS)	`'ANALYTICAL'`	529.15026E+02	1.0E-10

4 Modeling B

4.1 Characteristics of modeling

This test of the criteria CAS-tests `MATAKE_MODI_AV`, `DANG_VAN_MODI_AV`, `FATESOCI_MODI_AV`, criterion formulates some for the loading not-periodical and biaxial and proportional;

4.2 Characteristics of the grid

Identical to modeling A.

4.3 Sizes tested and results

- **Criterion `'MATAKE_MODI_AV'` and associated criterion in formula**

For the results **with the node** $N206$ and with **mesh** $M60$ **(not Gauss 3)**
The option `COURBE_GRD_VIE=' WOHLER'` and L
The option `COURBE_GRD_VIE = 'FORMES_VIE'` and `FORMULE_VIE = WHOL`:

Identification	Type of reference	Value of reference
component x of n_1 and n_2	`'AUTRE_ASTER'`	-0.38268343236509 0.38268343236509
component y of n_1 and n_2	`'AUTRE_ASTER'`	0.92718385456679 0.92387953251129
component z of n_1 and n_2	`'AUTRE_ASTER'`	0.0000000000000E+00
$ENDO(n_1)$	`'AUTRE_ASTER'`	7.0532362250863E-04

In the table above, the components x and y of n_1 and n_2 two values have because there exist two vectors which correspond to the same value of damage $ENDO(n_1) = ENDO(n_2)$.

The option `COURBE_GRD_VIE= 'FORMES_VIE'` and `FORMULE_VIE = WHOL_F`:

Identification	Type of reference	Value of reference
component x of n_1 and n_2	`'AUTRE_ASTER'`	-0.38268343236509 0.38268343236509
component y of n_1 and n_2	`'AUTRE_ASTER'`	0.92718385456679 0.92387953251129
component z of n_1 and n_2	`'AUTRE_ASTER'`	0.0000000000000E+00
$ENDO(n_1)$	`'AUTRE_ASTER'`	3.3180845213285E-04

In the table above, the components x and y of n_1 and n_2 two values have because there exist two vectors which correspond to the same value of damage $ENDO(n_1) = ENDO(n_2)$.

- **Criterion `'DANG_VAN_MODI_AV'` and associated criterion in formula**

For the results **with the node** $N206$ and with **mesh** $M60$ **(not Gauss 3)**
Option `COURBE_GRD_VIE=' WOHLER'` and
The option `COURBE_GRD_VIE=' FORMES_VIE'` and `FORMULE_VIE = WHOL`:

Identification	Type of reference	Value of reference
component x of n_1 and n_2	'AUTRE_ASTER'	-7.0710678118655E-01 7.0710678118655E-01
component y of n_1 and n_2	'AUTRE_ASTER'	7.0710678118655E-01
component z of n_1 and n_2	'AUTRE_ASTER'	0.0000000000000E+00
$ENDO(n_1)$	'AUTRE_ASTER'	1.3419917535855E-04

In the table above, the components x and y of n_1 and n_2 two values have because there exist two vectors which correspond to the same value of damage $ENDO(n_1)$ = $ENDO(n_2)$.

The option COURBE_GRD_VIE= 'FORMES_VIE' and FORMULE_VIE = WHOL_F:

Identification	Type of reference	Value of reference
component x of n_1 and n_2	'AUTRE_ASTER'	-7.0710678118655E-01 7.0710678118655E-01
component y of n_1 and n_2	'AUTRE_ASTER'	7.0710678118655E-01
component z of n_1 and n_2	'AUTRE_ASTER'	0.0000000000000E+00
$ENDO(n_1)$	'AUTRE_ASTER'	8.7960237413997E-05

In the table above, the components x and y of n_1 and n_2 two values have because there exist two vectors which correspond to the same value of damage $ENDO(n_1)$ = $ENDO(n_2)$.

- **Criterion 'FATESOCI_MODI_AV' and associated criterion in formula:**

For the results **with the node** $N1$ and with **mesh** $M60$ **(not Gauss 3)**
The option COURBE_GRD_VIE=' WOHLER' and
The option COURBE_GRD_VIE=' FORMES_VIE' and FORMULE_VIE = MANCO1:

Identification	Type of reference	Value of reference
component x of n_1 and n_2	'AUTRE_ASTER'	-0.43051109680829 0.43051109680830
component y of n_1 and n_2	'AUTRE_ASTER'	0.90258528434986
component z of n_1 and n_2	'AUTRE_ASTER'	0
$ENDO(n_1)$	'AUTRE_ASTER'	0.43649132038876

In the table above, the component x of n_1 and n_2 has two values because there exist two vectors which correspond to the same value of damage $ENDO(n_1)$ = $ENDO(n_2)$.

5 Modeling C

5.1 Characteristics of modeling

The criterion in formula makes it possible to find the criteria `'MATAKE_MODI_AC'` and `'DANG_VAN_MODI_AV'` and of the criterion in formula associated for the loading multiaxial.

5.2 Characteristics of the grid

The grid is identical to that of modeling A.

5.3 Sizes tested and results

For periodic loading:
For the results with **mesh** $M60$ **(not Gauss 3)** for the option `COURBE_GRD_VIE = 'WOHLER'` and for the option `COURBE_GRD_VIE = 'FORMES_VIE'` and `FORMULE_VIE = WHOL`:

Identification	Type of reference	Value of reference
$\Delta\tau(n_1)$	'NON_REGRESSION'	1 . 330171E + 02
component x of n_1	'NON_REGRESSION'	6.972459E – 02
component y of n_1	'NON_REGRESSION'	9.969556E – 01
component z of n_1	'NON_REGRESSION'	– 3.489950E – 02
$N_{\max}(n_1)$	'NON_REGRESSION'	2.357226E+00
$N_m(n_1)$	'NON_REGRESSION'	2.220625E – 14
$\varepsilon_{\max}(n_1)$	'NON_REGRESSION'	0.000000 E + 00
$\varepsilon_m(n_1)$	'NON_REGRESSION'	– 3.627373E – 05
$\sigma_{eq}(n_1)$	'NON_REGRESSION'	2.348841E+02
$Nb_{cr}(n_1)$	'NON_REGRESSION'	2.583800E+04
$ENDO(n_1)$	'NON_REGRESSION'	3.870305E – 05
$\Delta\tau(n_2)$	'NON_REGRESSION'	1.330158E + 02
component x of n_2	'NON_REGRESSION'	– 9.901402E – 01
component y of n_2	'NON_REGRESSION'	6.906669E – 02
component z of n_2	'NON_REGRESSION'	1.218693E – 01
$N_{\max}(n_2)$	'NON_REGRESSION'	1.264927E+02
$N_m(n_2)$	'NON_REGRESSION'	1.581158E +0 1
$\varepsilon_{\max}(n_2)$	'NON_REGRESSION'	6.474850E – 04

$\varepsilon_m(n_2)$	'NON_REGRESSION'	8.093563E – 05
$\sigma_{eq}(n_2)$	'NON_REGRESSION'	3.892627E+02
$Nb_{cr}(n_2)$	'NON_REGRESSION'	3.323100E+04
$ENDO(n_2)$	'NON_REGRESSION'	3.009210E – 05

For the results with **mesh** $M60$ **(not Gauss 3)** for the option COURBE_GRD_VIE = 'FORMES_VIE' and FORMULE_VIE = WHOL_F:

Identification	Type of reference	Value of reference
$\Delta\tau(n_1)$	'NON_REGRESSION'	1 . 330171E + 02
component x of n_1	'NON_REGRESSION'	6.972459E – 02
component y of n_1	'NON_REGRESSION'	9.969556E – 01
component z of n_1	'NON_REGRESSION'	– 3.489950E – 02
$N_{\max}(n_1)$	'NON_REGRESSION'	2.357226E+00
$N_m(n_1)$	'NON_REGRESSION'	2.220625E – 14
$\varepsilon_{\max}(n_1)$	'NON_REGRESSION'	0.000000 E + 00
$\varepsilon_m(n_1)$	'NON_REGRESSION'	– 3.627373E – 05
$\sigma_{eq}(n_1)$	'NON_REGRESSION'	2.348841E+02
$Nb_{cr}(n_1)$	'NON_REGRESSION'	5.1477E+04
$ENDO(n_1)$	'NON_REGRESSION'	1.9426163934314E-05
$\Delta\tau(n_2)$	'NON_REGRESSION'	1.330158E + 02
component x of n_2	'NON_REGRESSION'	– 9.901402E – 01
component y of n_2	'NON_REGRESSION'	6.906669E – 02
component z of n_2	'NON_REGRESSION'	1.218693E – 01
$N_{\max}(n_2)$	'NON_REGRESSION'	1.264927E+02
$N_m(n_2)$	'NON_REGRESSION'	1.581158E +0 1
$\varepsilon_{\max}(n_2)$	'NON_REGRESSION'	6.474850E – 04
$\varepsilon_m(n_2)$	'NON_REGRESSION'	8.093563E – 05
$\sigma_{eq}(n_2)$	'NON_REGRESSION'	3.892627E+02
$Nb_{cr}(n_2)$	'NON_REGRESSION'	5.1477E+04
$ENDO(n_2)$	'NON_REGRESSION'	1.9426163934314E-05

For the results **with the node** $N214$ for the option `COURBE_GRD_VIE=' WOHLER'` and for the option `COURBE_GRD_VIE = 'FORMES_VIE'` and `FORMULE_VIE = WHOL`:

Identification	Type of reference	Value of reference
$\Delta\tau(n_1)$	'NON_REGRESSION'	1.1557902030140E+02
component x of n_1	'NON_REGRESSION'	3.8280107156988E-01
component y of n_1	'NON_REGRESSION'	8.4447216637038E-01
component z of n_1	'NON_REGRESSION'	3.7460659341591E-01
$N_{max}(n_1)$	'NON_REGRESSION'	7.3701737537055E+01
$N_m(n_1)$	'NON_REGRESSION'	-6.6290480086559E+00
$\varepsilon_{max}(n_1)$	'NON_REGRESSION'	0.
$\varepsilon_m(n_1)$	'NON_REGRESSION'	-4.2254262706848E-05
$\sigma_{eq}(n_1)$	'NON_REGRESSION'	2.8392113675768E+02
$Nb_{cr}(n_1)$	'NON_REGRESSION'	1.3505000000000E+04
$ENDO(n_1)$	'NON_REGRESSION'	7.4047664409136E-05
$\Delta\tau(n_2)$	'NON_REGRESSION'	1.1520977056656E+02
component x of n_2	'NON_REGRESSION'	-9.1924333354254E-01
component y of n_2	'NON_REGRESSION'	3.9019564505737E-01
component z of n_2	'NON_REGRESSION'	5.2335956242944E-02
$N_{max}(n_2)$	'NON_REGRESSION'	1.1296755026397E+02
$N_m(n_2)$	'NON_REGRESSION'	6.8110853707598E+00
$\varepsilon_{max}(n_2)$	'NON_REGRESSION'	3.6085283407484E-04
$\varepsilon_m(n_2)$	'NON_REGRESSION'	4.5106604259354E-05
$\sigma_{eq}(n_2)$	'NON_REGRESSION'	3.4226598124581E+02
$Nb_{cr}(n_2)$	'NON_REGRESSION'	4.8720000000000E+03
$ENDO(n_2)$	'NON_REGRESSION'	2.0525129321838E-04

For the results **with the node** $N214$ for the option `COURBE_GRD_VIE=' FORMES_VIE'` and `FORMULE_VIE = WHOL_F`:

Identification	Type of reference	Value of reference

$\Delta\tau(n_1)$	'NON_REGRESSION'	1.1557902030140E+02
component x of n_1	'NON_REGRESSION'	3.8280107156988E-01
component y of n_1	'NON_REGRESSION'	8.4447216637038E-01
component z of n_1	'NON_REGRESSION'	3.7460659341591E-01
$N_{\max}(n_1)$	'NON_REGRESSION'	7.3701737537055E+01
$N_m(n_1)$	'NON_REGRESSION'	-6.6290480086559E+00
$\varepsilon_{\max}(n_1)$	'NON_REGRESSION'	0.
$\varepsilon_m(n_1)$	'NON_REGRESSION'	-4.2254262706848E-05
$\sigma_{eq}(n_1)$	'NON_REGRESSION'	2.8392113675768E+02
$Nb_{cr}(n_1)$	'NON_REGRESSION'	2.0270E+04
$ENDO(n_1)$	'NON_REGRESSION'	4.933357265479E-05
$\Delta\tau(n_2)$	'NON_REGRESSION'	1.1520977056656E+02
component x of n_2	'NON_REGRESSION'	-9.1924333354254E-01
component y of n_2	'NON_REGRESSION'	3.9019564505737E-01
component z of n_2	'NON_REGRESSION'	5.2335956242944E-02
$N_{\max}(n_2)$	'NON_REGRESSION'	1.1296755026397E+02
$N_m(n_2)$	'NON_REGRESSION'	6.8110853707598E+00
$\varepsilon_{\max}(n_2)$	'NON_REGRESSION'	3.6085283407484E-04
$\varepsilon_m(n_2)$	'NON_REGRESSION'	4.5106604259354E-05
$\sigma_{eq}(n_2)$	'NON_REGRESSION'	3.4226598124581E+02
$Nb_{cr}(n_2)$	'NON_REGRESSION'	1.01240E+04
$ENDO(n_2)$	'NON_REGRESSION'	9.8775850589871E-05

For loading not-periodical:

For the results **with the node** $N214$ for the option COURBE_GRD_VIE=' WOHLER' and for the option COURBE_GRD_VIE=' FORMES_VIE' and FORMULE_VIE = WHOL:

Identification	Type of reference	Value of reference
component x of n_1	'NON_REGRESSION'	3.8280107156988E-01
component y of n_1	'NON_REGRESSION'	8.4447216637038E-01
component z of n_1	'NON_REGRESSION'	3.7460659341591E-01

$ENDO(n_1)$	'NON_REGRESSION'	9.2779623136707E-05

For the results **with the node** $N206$ and with **mesh** $M60$ **(not Gauss 3)** for the option COURBE_GRD_VIE= 'FORMES_VIE' and FORMULE_VIE = WHOL_F:

Identification	Type of reference	Value of reference
component x of n_1	'NON_REGRESSION'	3.8280107156988E-01
component y of n_1	'NON_REGRESSION'	8.4447216637038E-01
component z of n_1	'NON_REGRESSION'	3.7460659341591E-01
$ENDO(n_1)$	'NON_REGRESSION'	6.1692384350833E-05

6 Modeling D

6.1 Characteristics of modeling

The features tested are new sizes (which are not part of the criteria existing already tested in other modelings). Only the option CRITERE=' FORMULE_CRITERE' order CALC_FATIGUE and the curve of life called by the name ‘WOHLER‘ are used.

It is noted that the behavior is elastoplastic and the loading is uniaxial and periodic.

6.2 Characteristics of the grid

The grid is identical to that of modeling A.

6.3 Sizes tested and results

For the results **with the node** $N1$ and with **mesh** $M60$:

Identification	Type of reference	Value of reference
‘DEPSPE’	‘ANALYTICAL’	7.5E-4
‘EPSPR1’	‘ANALYTICAL’	7.625E-4
‘SIGNM1’	‘ANALYTICAL’	200
‘APHYDR’	‘ANALYTICAL’	66.6666
‘DENDIS’	‘ANALYTICAL’	0.45
‘DENDIE’	‘ANALYTICAL’	0.173333
‘DSIGEQ’	‘ANALYTICAL’	200
‘EPSNM1’	‘ANALYTICAL’	1.75E-3
‘INVA2S’	‘ANALYTICAL’	1.616666E-3
‘DSITRE’	‘ANALYTICAL’	50
‘DEPTRE’	‘ANALYTICAL’	6.0625E-4
‘DEPTRE’	‘ANALYTICAL’	3.67423E-3
‘DEPSEE’	‘ANALYTICAL’	0.00086666666

7 Modeling E

7.1 Characteristics of modeling

The features tested are new sizes.
It is noted that the behavior is elastoplastic and the loading is biaxial and not-periodical.

7.2 Characteristics of the grid

The grid is identical to that of modeling A.

7.3 Sizes tested and results

The value of reference corresponds to the endommagemenT (`ENDO1`) and the results were got **with the node** $N1$ and with **mesh** $M60$ via **formula of Basquin** :

Identification	**Type of reference**	**Value of reference**
Criteria		
$\frac{\lvert SIPR1-SIPR2\rvert}{2}$	`'ANALYTICAL'`	1.0707149E-03
$\frac{\lvert SITN1-SITN2\rvert}{2}$	`'ANALYTICAL'`	1.0707149E-03
$\frac{SIPN1-SIPN2}{2}$	`'ANALYTICAL'`	1.0707149E-03
$\frac{SIGEQ1-SIGEQ2}{2}$	`'ANALYTICAL'`	4.287285E-03

The value of reference always corresponds to the damage (`ENDO1`) and the results were got **with node** $N1$ and with **mesh** $M60$ with one **interpolation** curve of Wöhler :

Identification	**Type of reference**	**Value of reference**
Criteria		
$\frac{\lvert SIPR1-SIPR2\rvert}{2}$	`'ANALYTICAL'`	1.9212572E-03
$\frac{\lvert SITN1-SITN2\rvert}{2}$	`'ANALYTICAL'`	1.9212572E-03
$\frac{SIPN1-SIPN2}{2}$	`'ANALYTICAL'`	1.9212572E-03
$\frac{SIGEQ1-SIGEQ2}{2}$	`'ANALYTICAL'`	5.8175699E-03

8 Modeling F

8.1 Characteristics of modeling

The features tested are new sizes.
It is noted that various behaviors and loading are tested: rubber band, biaxial and elastoplastic, uniaxial not-periodical then and not-periodical.

8.2 Characteristics of the grid

The grid is identical to that of modeling A.

8.3 Sizes tested and results

- Result got with the first loading (**SOL_NL**):
The value of reference corresponds to the damage (ENDO1) and the results were got **with the node** $N1$ and with **mesh** $M60$ via **formula of Basquin** :

Identification	Type of reference	Value of reference
Criteria		
$\frac{\lvert EPSN1-EPSN2\rvert}{2}$	'ANALYTICAL'	1.08363973E-05
$\frac{\lvert ETPR1-ETPR2\rvert}{2}$	'ANALYTICAL'	1.0 8363973 E-0 5
$\frac{\lvert ETEQ1-ETEQ2\rvert}{2}$	'ANALYTICAL'	1.06338423E-05

The value of reference always corresponds to the damage (ENDO1) and the results were got **with node** $N1$ and with **mesh** $M60$ with one **interpolation** curve of Wöhler :

Identification	Type of reference	Value of reference
Criteria		
$\frac{\lvert EPSN1-EPSN2\rvert}{2}$	'ANALYTICAL'	3.26558686E-05
$\frac{\lvert ETPR1-ETPR2\rvert}{2}$	'ANALYTICAL'	3.26558686E-05
$\frac{\lvert ETEQ1-ETEQ2\rvert}{2}$	'ANALYTICAL'	3.21404432E-05

- Result got with the second loading (**SOL_NL2**):
  The value of reference corresponds to the damage (ENDO1) and the results were got **with the node** $N1$ and with **mesh** $M60$ via **formula of Basquin** :

Identification	Type of reference	Value of reference
Criteria		
$\frac{\vert EPSN1-EPSN2\vert}{2}$	'ANALYTICAL'	1.449229E-04
$\frac{\vert ETPR1-ETPR2\vert}{2}$	'ANALYTICAL'	1. 449229 E-0 4
$\frac{\vert ETEQ1-ETEQ2\vert}{2}$	'ANALYTICAL'	6.5320499E-05

The value of reference always corresponds to the damage (ENDO1) and the results were got **with node** $N1$ and with **mesh** $M60$ with one **interpolation** curve of Wöhler :

Identification	Type of reference	Value of reference
Criteria		
$\frac{\vert EPSN1-EPSN2\vert}{2}$	'ANALYTICAL'	2.408735E-04
$\frac{\vert ETPR1-ETPR2\vert}{2}$	'ANALYTICAL'	2.408735 E-0 4
$\frac{\vert ETEQ1-ETEQ2\vert}{2}$	'ANALYTICAL'	1.322816E-04

- Result got with the third loading (**SOL_NL3**):
  The value of reference corresponds to the damage (ENDO1) and the results were got **with the node** $N1$ and with **mesh** $M60$ via **formula of Basquin** :

Identification	Type of reference	Value of reference
Criteria		
$\frac{\vert EPPR1-EPPR2\vert}{2}$	'ANALYTICAL'	1.377855E-02

The value of reference always corresponds to the damage (ENDO1) and the results were got **with node** $N1$ and with **mesh** $M60$ with one **interpolation** curve of Wöhler :

9 Modeling G

9.1 Characteristics of modeling

The features tested are new sizes.
It is noted that various behaviors and loading are tested: an elastic material and an elastoplastic material.

9.2 Characteristics of the grid

The grid is identical to that of modeling A.

9.3 Sizes tested and results

- **Criteria of `DANG_VAN _MODI_AC`, of `MATAKE_MODI_AC`, of `DANG_VAN_MODI_AV`**

For the results of ϕ_z with the node $N1$ for an elastic material.

Value of α	Type of reference	Value of reference
−1,-0.5,0..10	'ANALYTICAL'	45

For the results of ϕ_z with the node $N1$ for an elastoplastic material.

Value of α	Type of reference	Value of reference
0,1,2,3,4	'ANALYTICAL'	45

- Criterion of **`MATAKE_MODI_AV`**

Value of α	Type of reference	Value of reference
−1	'ANALYTICAL'	45
−0.5	'ANALYTICAL'	45,72
0	'ANALYTICAL'	46,43
0,5	'ANALYTICAL'	47,14
1	'ANALYTICAL'	47,86
1,5	'ANALYTICAL'	48,56
2	'ANALYTICAL'	49,27
2,5	'ANALYTICAL'	49,96
3	'ANALYTICAL'	50,65
3,5	'ANALYTICAL'	51,34
4	'ANALYTICAL'	52,02
4,5	'ANALYTICAL'	52,69
5	'ANALYTICAL'	53,35
5,5	'ANALYTICAL'	54
6	'ANALYTICAL'	54,65
6,5	'ANALYTICAL'	55,28

7	'ANALYTICAL'	55,9
7,5	'ANALYTICAL'	56,51
8	'ANALYTICAL'	57,11
8,5	'ANALYTICAL'	57,7
9	'ANALYTICAL'	58,28
9,5	'ANALYTICAL'	58,85
10	'ANALYTICAL'	59,41

For the results of ϕ_z **with the node** $N1$ for an elastoplastic material.

Value of α	Type of reference	Value of reference
0	'ANALYTICAL'	46,43
1	'ANALYTICAL'	47,86
2	'ANALYTICAL'	49,27
3	'ANALYTICAL'	50,65
4	'ANALYTICAL'	52,02

- **Criterion of FATESOCI_MODI_AV**

For the results of ϕ_z **with the node** $N1$ for an elastic material.

Value of α	Type of reference	Value of reference
−1	'ANALYTICAL'	45
−0.5	'ANALYTICAL'	45,34
0	'ANALYTICAL'	45,67
0,5	'ANALYTICAL'	45,99
1	'ANALYTICAL'	46,31
1,5	'ANALYTICAL'	46,61
2	'ANALYTICAL'	46,91
2,5	'ANALYTICAL'	47,2
3	'ANALYTICAL'	47,48
3,5	'ANALYTICAL'	47,75
4	'ANALYTICAL'	48,01
4,5	'ANALYTICAL'	48,27
5	'ANALYTICAL'	48,51
5,5	'ANALYTICAL'	48,75
6	'ANALYTICAL'	48,98
6,5	'ANALYTICAL'	49,2
7	'ANALYTICAL'	49,42

7,5	'ANALYTICAL'	49,63
8	'ANALYTICAL'	49,83
8,5	'ANALYTICAL'	50,03
9	'ANALYTICAL'	50,22
9,5	'ANALYTICAL'	50,4
10	'ANALYTICAL'	50,58

For the results of ϕ_z **with the node** $N1$ for an elastoplastic material.

Value of α	Type of reference	Value of reference
0	'ANALYTICAL'	45,67
1	'ANALYTICAL'	46,31
2	'ANALYTICAL'	46,91
3	'ANALYTICAL'	47,48
4	'ANALYTICAL'	48,01

10 Modeling H

10.1 Characteristics of modeling

The features tested are new sizes and the keyword FORMULE_CRITIQUE. Only the option CRITERE=' FORMULE_CRITERE' order CALC_FATIGUE and the curves of life called by the formulas are used.

It is noted that the behavior is elastoplastic and the loading is uniaxial and periodic.

10.2 Characteristics of the grid

The grid is identical to that of modeling A.

10.3 Sizes tested and results

For the results **with the node** $N1$ and with **mesh** $M60$:

Identification	Type of reference	Value of reference
FORMULE_CRITIQUE = 'DTAUCR' or 'MTAUCR'		
'SIGEQ1'	'ANALYTICAL'	100 MPa
'ENDO1'	'ANALYTICAL'	1.028E-6
'NBRUP1'	'ANALYTICAL'	9.73E5
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(0,707, -0,707), 0,707.0
FORMULE_CRITIQUE = 'DGAMCR' or 'MGAMCR'		
'SIGEQ1'	'ANALYTICAL'	9.73E5
'ENDO1'	'ANALYTICAL'	1.583E-4
'NBRUP1'	'ANALYTICAL'	6.3163E3
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(0,707, -0,707), 0,707.0
FORMULE_CRITIQUE = 'DSINCR' or 'MSINCR'		
'SIGEQ1'	'ANALYTICAL'	200
'ENDO1'	'ANALYTICAL'	1.348E-5
'NBRUP1'	'ANALYTICAL'	7.418E4
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(- 1.1), 0.0174,0
FORMULE_CRITIQUE = 'DEPNCR' or 'MEPNCR'		
'SIGEQ1'	'ANALYTICAL'	1.75E-3
'ENDO1'	'ANALYTICAL'	2.11E-5
'NBRUP1'	'ANALYTICAL'	4.74E4
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(- 1, 1), 0.0174,0

FORMULE_CRITIQUE = 'DGAMPC' or 'MGAMPC'		
'SIGEQ1'	'ANALYTICAL'	1.125E-3
'ENDO1'	'ANALYTICAL'	1.3782E-6
'NBRUP1'	'ANALYTICAL'	7.256E5
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(0,707, -0,707), 0,707.0
FORMULE_CRITIQUE = 'DEPNPC' or 'MEPNPC'		
'SIGEQ1'	'ANALYTICAL'	0.75E-3
'ENDO1'	'ANALYTICAL'	1.126E-7
'NBRUP1'	'ANALYTICAL'	8.88E6
'VNM1X', 'VNM1Y', 'VNM1Z'	'ANALYTICAL'	(- 1.1), 0.0174,0

11 Modeling I

See Modeling F (left 8).

12 Summary of the results

The got results are in perfect agreement with the reference solution for modeling A. modeling B does not have reference solutions associated with the criteria. Modeling C does not have a reference solution, it acts of a test of not-regression.

Results of modelings D, E, F, G, H and I agree with the analytical results.