

## RCCM09 - Operator POST\_RCCM : calculation of the factor of starting

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### Summary:

This test of thermomechanical linear statics makes it possible to validate the calculation of the factor of starting with the order `POST_RCCM` (option `STARTING`, results of the type `EVOLUTION`).

The analytical solution is simple, and makes it possible to test postprocessing within the meaning of the `RCC_M`.

Modeling A of the case test consists of a set of unit tests, where the constraints are directly provided and not calculated, to check the good performance of the calculation of the factor of starting.

Modeling B corresponds to the complete resolution (modeling, calculation of the constraints and estimate of the factors of starting) of a problem of reference presented hereafter.

## 1 Problem of reference of modeling B

### 1.1 Geometry

The problem of reference is plan. One considers a rectangular plate, of width  $W$ , comprising an emerging crack length  $a$ .

One is interested in the factor of starting related to the geometrical singularity in  $P$ , the structure being subjected to several transitory mechanical loadings.

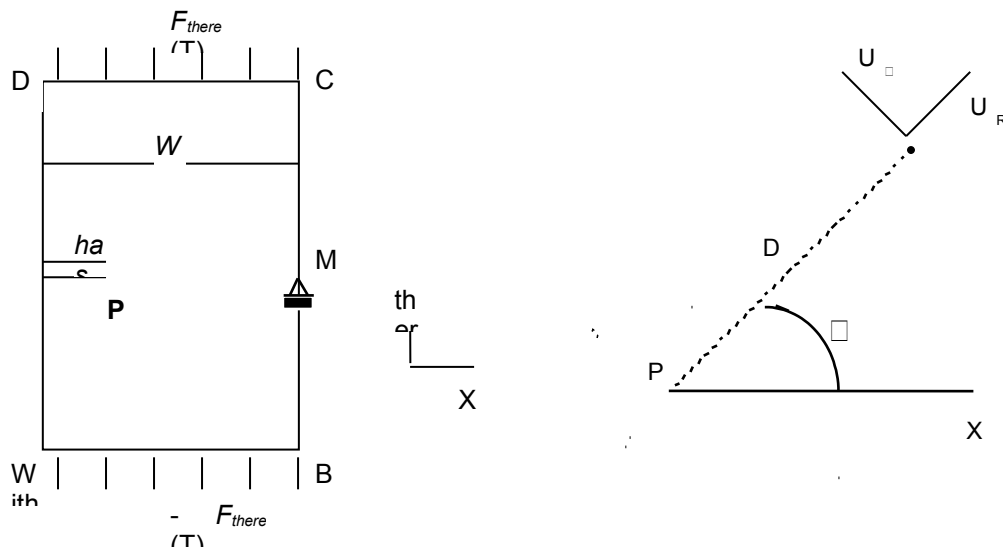


Figure 1.1-a : Definition of the problem of reference and the local reference mark

### 1.2 Properties materials

The properties material selected are the following ones:

- 1) module of Young:  $E = 2. E + 05 \text{ MPa}$  ;
- 2) Poisson's ratio:  $\nu = 0,3$  .

The characteristics suitable for the calculation of the factor of starting to direction RCC-M correspond to the case of an alloy Ni-Cr-Fe (Inconel):

- 1) parameters of the law of starting:  $A_{AMORC} = 7091$  and  $B_{AMORC} = -0.1823$ ;
- 2) parameter of the effective constraint:  $R_{AMORC} = 1.63$ ;
- 3) distance D with the singularity:  $D_{AMORC} = 0,046 \text{ mm}$  .

The law of starting is expressed in the following way:

$$\Delta\sigma_{eff}(d) = A_{AMORC} \cdot (N_a)^{B_{AMORC}}$$

with  $NR_{has}$  the number of acceptable cycles and by taking the following relation between amplitude of real variation of the constraints  $\Delta\sigma_{\theta\theta}$  and effective  $\Delta\sigma_{eff}$  :

$$\Delta\sigma_{eff} = \frac{\Delta\sigma_{\theta\theta}}{1 - R/R_{AMORC}}, \text{ with } R \text{ report of load.}$$

### 1.3 Boundary conditions and loadings

The structure is subjected to two transitory mechanical loadings; all the loadings consist of a linear force  $F_y$  imposed on the segments  $AB$  and  $CD$ .

The rigid movements of body are blocked at the point Mr.

Transient 1:

$$t=0s \quad |F_y|=0 \quad ; \quad t=1s \quad |F_y|=100 f^0$$

Transient 2:

$$t=0s \quad |F_y|=10 f^0 \quad ; \quad t=1s \quad |F_y|=110 f^0 \quad ; \quad t=2s \quad |F_y|=50 f^0$$

where  $f^0 = 3 N.mm^{-1}$ .

## 2 Reference solution

### 2.1 Results of reference

#### 2.1.1 Calculation of the factors of intensity of the constraints

The reference solution for an emerging crack of depth  $has$  in a plate thickness  $W$  solicated in mode  $I$  pure (force  $\sigma$  along the axis  $y$ ) is the following one:

$$K_I = Y \sigma \sqrt{a}, \text{ with } Y = 1.99 - 0.41 \frac{a}{W} + 18.7 \left( \frac{a}{W} \right)^2 - 38.48 \left( \frac{a}{W} \right)^3 + 53.85 \left( \frac{a}{W} \right)^4$$

*Digital application:* for a crack of  $15 \text{ mm}$  of depth ( $W = 72 \text{ mm}$ ) and the loading  $f^0$ , one obtains  $K_I = 2,874 \text{ MPa} \cdot \sqrt{\text{mm}}$ .

#### 2.1.2 Calculation of the constraints around the bottom of crack

The analytical solution for the constraint on a circle of radius  $r$  around the bottom of crack, obtained for a crack in infinite medium, is the following one:

$$\sigma_{\theta\theta} = \frac{K_I}{4\sqrt{2\pi r}} \left( 3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right)$$

*Digital application:* for the loading  $f^0$  and for a ray  $r = R_{AMORC} = 0,046 \text{ mm}$ , one obtains:

$$\theta = 0 : \sigma_{\theta\theta} = 53.5 \text{ MPa} \quad ; \quad \theta = \pi/2 : \sigma_{\theta\theta} = 18.9 \text{ MPa}$$

#### 2.1.3 Calculation of the factor of starting

The factor of starting associated with the geometrical singularity in  $P$  is calculated analytically while following the procedure prescribed in the RCC-M [3]. Every provided moment is regarded as local extrema and is thus appointed in calculation.

One notes  $s^0$  the value of  $\sigma_{\theta\theta}$  raised with a ray  $r$  bottom of crack, with an angle  $\theta$  given for the loading  $f^0$ . One can then build the table of the amplitudes of variation of  $\sigma_{\theta\theta}$  for all the possible combinations:

			Transient 1		Transient 2		
			$Nb_{occur} = i$		$Nb_{occur} = j$		
			$t=0$	$t=1$	$t=0$	$t=1$	$t=2$
Transient 1	$Nb_{occur} = i$	$t=0$		$100 s^\circ$	$10 s^\circ$	$110 s^\circ$	$50 s^\circ$
		$t=1$			$90 s^\circ$	$10 s^\circ$	$50 s^\circ$
Transient 2	$Nb_{occur} = j$	$t=0$				$100 s^\circ$	$40 s^\circ$
		$t=1$					$60 s^\circ$
		$t=2$					

The taking into account of the report of load  $R$  of each combination allows to calculate the amplitude of variation of the effective constraints  $\Delta \sigma_{eff}$ :

			Transient 1		Transient 2		
			$Nb_{occur} = i$		$Nb_{occur} = j$		
			$t=0$	$t=1$	$t=0$	$t=1$	$t=2$
Transient 1	$Nb_{occur} = i$	$t=0$		$100 s^\circ$	$10 s^\circ$	$110 s^\circ$	$50 s^\circ$
		$t=1$			$95,6 s^\circ$	$22,6 s^\circ$	$72,1 s^\circ$
Transient 2	$Nb_{occur} = j$	$t=0$				$105,9 s^\circ$	$45,6 s^\circ$
		$t=1$					$83,6 s^\circ$
		$t=2$					

The calculation of the factor of starting  $FA$  be carried out according to an iterative process:

- 1) identification of the transient of number of occurrences not no one leading to the maximum of  $\Delta\sigma_{eff}$  ;
- 2) calculation of the elementary factor of associated starting, using the law of tiredness;
- 3) reactualization amongst occurrences of the treated combination.

Two cases are successively studied with the numbers of occurrences of the two following transients:  $i=j=1$  and  $i=1 ; j=2$ . One notes  $N_{kl-mn}$  the number of acceptable cycles for the combination of  $l$ -ième step of time of  $k$ -ième transient and of  $n$ -ième step of time of  $m$ -ième transient (calculated with part of  $\Delta\sigma_{eff}$  of this combination and law of tiredness).

$$\text{1st case: } FA_1 = \frac{1}{N_{11-22}} + \frac{1}{N_{12-21}} + \frac{1}{N_{22-23}}$$

$$\text{2nd case: } FA_2 = \frac{1}{N_{11-22}} + \frac{1}{N_{11-12}} + \frac{1}{N_{12-21}} + \frac{1}{N_{22-23}}$$

Digital application: for modeling A of this case test, one supposes that  $s^\circ = 1$ . One thus has:

$$FA_1 = 2,005 \cdot 10^{-10} ; \quad FA_2 = 2,710 \cdot 10^{-10}$$

For modeling B, the value of  $s^\circ$  depends on the angle  $\theta$ , and can be calculated using the formula indicated in the paragraph 2.1.2.

$$\theta = 0 : \quad \sigma^\circ = 53,5 \text{MPa} , \quad FA_1 = 0,607 \quad ; \quad FA_2 = 0,819$$

$$\theta = 90^\circ : \quad \sigma^\circ = 18,9 \text{MPa} , \quad FA_1 = 2,01 \cdot 10^{-3} ; \quad FA_2 = 2,72 \cdot 10^{-3}$$

## 2.2 Bibliographical references

- [1] W.F. BROWN, J.E. STRAWLEY: "Planes Strain Ace Toughness Testing of High Strength Material", American Society of Testing and Materials, STP 410
- [2] J.B. LEBLOND: "Breaking process fragile and ductile", Lavoisier, Paris, 2003.
- [3] RCC-M: "Breaking process fragile and ductile", Lavoisier, Paris, 2003.

## 3 Modeling A

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### 3.1 Characteristics of modeling

Modeling A only aims at validating the features of calculation of the factor of starting in the operator POST\_RCCM starting from the constraints directly given.

There is thus in this modeling neither grid, nor mechanical calculation.

### 3.2 Values tested and results

One tests the calculation of the factor of starting with:

- various cases of constraints min and max producing the same amplitude of variation (100);
- a producing case of combination of 2 transients of the different constraints;
- a case of negative values;
- a case with a number of occurrences.

The values of the factor of starting calculated are exactly equal to the analytical values of reference.

## 4 Modeling B

### 4.1 Characteristics of modeling

The objective of modeling B is to validate the whole of the process of the calculation of the factor of starting, of the resolution of the mechanical problem to postprocessing with POST\_RCCM.

Calculation is carried out on a modeling 2D plane deformations, with a material with the linear elastic behavior.

### 4.2 Characteristics of the grid

Many nodes: 30000

Many meshes: 10000 quadratic meshes (SEG3, TRIA6 and QUAD8)

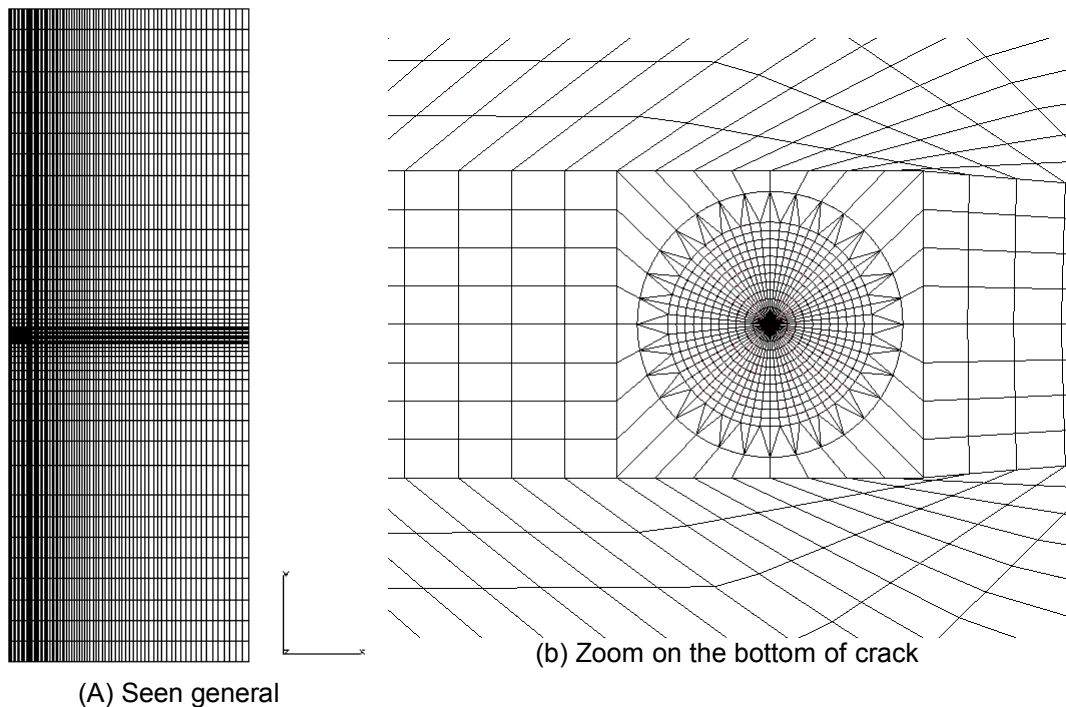


Figure 4.2-a : Grid – case test rccm09b

### 4.3 Values tested and results

The various values tested are gathered in the table below.

The good precision on the factor of intensity of the constraints shows that the grid used is sufficiently fine to collect the singularity of displacements in bottom of crack.

The comparison of the constraints  $\sigma_{\theta\theta}$  according to the angle  $\theta$  is also satisfactory: the variation on the maximum value of  $\sigma_{\theta\theta}$ , obtained for  $\theta=0$ , is of 2%. The difference between analytical solution and digital solution can be explained by the effects of the edge of the structure on the stress field: one indeed compares an analytical solution obtained in an infinite medium with a calculation on a finished structure (crack of 15 mm of depth, plate of 72 mm of width).

The maximum factor of starting, calculated for  $\theta=0$ , is close to the analytical solution for the two studied cases. For  $\theta=90^\circ$ , the variation is important, like consequence of the variation already noted on the constraints.

Value tested	Reference	Aster	Variation
$K_I$ ( MPa. $\sqrt{mm}$ ) (method 3)	2.874	2.855	-0,6%
$\sigma_{\theta\theta}$ ( $\theta=0^\circ$ ) ( MPa )	53.5	54.5	2%
$\sigma_{\theta\theta}$ ( $\theta=90^\circ$ ) ( MPa )	18.9	17.5	-7%
$FA_1$ ( $\theta=0^\circ$ )	0.607	0.673	11%
$FA_1$ ( $\theta=90^\circ$ )	$2,01.10^{-3}$	$1,32.10^{-3}$	-34%
$FA_2$ ( $\theta=0^\circ$ )	0.819	0.909	11%
$FA_2$ ( $\theta=90^\circ$ )	$2,72.10^{-3}$	$1,786.10^{-3}$	-34%

**Note:**

The constraint  $\sigma_{\theta\theta}$  corresponds, in the table produced by MACR\_LIGN\_COUPE, with the component SIZZ.



## 5 Summary of the results

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This document presents two tests of checking of the calculation of the factor of starting in the operator `POST_RCCM` for results of the type `EVOLUTION`.

Modeling B particularly makes it possible to test the whole of calculation on a fissured plane structure: calculation of the constraints in the structure, extraction of these constraints on lines of circular cut with `MACR_LIGN_COUPE` and calculation of the factor of starting with `POST_RCCM`. The results are satisfactory, but underline the strong influence of the value of the constraints (expressed in the local reference mark) on the factor of calculated use.

One thus points out the need for using sufficiently fine grids (preferably radiant) in bottom of crack to collect correctly the singularity of constraints related to the presence of the geometrical singularity.