SSLV146 – Cubic full reinforced by reinforcements under triaxial loading

Summary

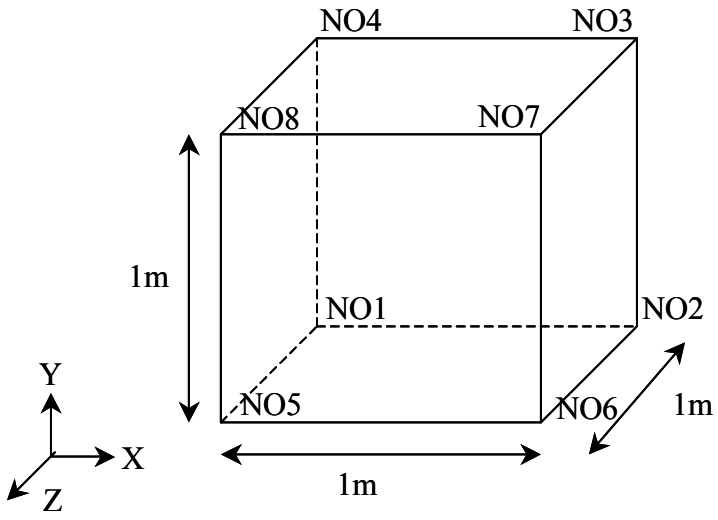
This test 3D enter within the framework of the validation of the formulation GRILLE_MEMBRANE. It is about a cube full of unit size. One places on each face a tablecloth of steel reinforcements so as to sweep all the possible directions. The loading consists into cubes displacements imposed on all the nodes of the structure.

The principal interest of this test is to test modeling GRILLE_MEMBRANE for various orientations and various elements (linear and quadratic). The results are compared with an analytical solution.

The units of all the digital values are in IF.

1 Problem of reference

1.1 Geometry



Six tablecloths of reinforcements (one by side of the cube) are defined:

- $GEOX$ (2 tablecloths): faces $NO1NO4NO8NO5$ and $NO2NO6NO7NO3$
- $GEOY$ (2 tablecloths): faces $NO1NO2NO6NO5$ and $NO4NO3NO7NO8$
- $GEOZ$ (2 tablecloths): faces $NO1NO2NO3NO4$ and $NO5NO6NO7NO8$

1.2 Properties of materials

For the full cube:

Modeling	A	B	C	D
$E(Pa)$	2	2	2E14	2E14
ν	0	0	0	0

For the tablecloths of reinforcements (any confused modeling)

$E=2\text{E}11\,Pa$, $\nu=0$

- Tablecloth $GEOX$: section per linear meter $0.01\,m^2/ml$, offsetting 0, orientation (ANGL_REP) $(30;0)$
- Tablecloth $GEOY$: section per linear meter $0.02\,m^2/ml$, offsetting 0, orientation (ANGL_REP) $(0;40)$
- Tablecloth $GEOZ$: section per linear meter $0.03\,m^2/ml$, offsetting 0, orientation (ANGL_REP) $(15\ ;70)$

1.3 Boundary conditions and loadings

The boundary conditions are the following ones:

$DX=0$ on the face $NO2NO3NO7NO6$
$DY=0$ on the face $NO1NO2NO6NO5$
$DZ=0$ on the face $NO1NO2NO3NO4$

The loading is applied in an increment in the following way (imposed displacements):

$DX=1$ on the face $NO1NO4NO8NO5$
$DY=2$ on the face $NO4NO3NO7NO8$
$DZ=3$ on the face $NO5NO6NO7NO8$

2 Reference solution

2.1 Formal solution

One seeks to define the deformation ε according to the principal direction of a tablecloth of reinforcement located in the plan $(x_1 ; y_1)$.

Taking into account the boundary conditions chosen, one can write:

$$\varepsilon = u_{x1}\cos^2(\theta) + u_{y1}\sin^2(\theta)$$

with (u_{x1}, u_{y1}) components of the vector displacement in the plan $(x_1 ; y_1)$ and θ the angle between the principal direction of the tablecloth of reinforcement and x_1.

To define the principal direction of the tablecloth, the nautical angles are used $(\alpha ; \beta)$ given by the keyword ANGL_REP. They define a vector v of which projection x_p on the tangent level of the tablecloth fixes the principal direction.

$$v = \cos(\alpha)\cos(\beta)\underline{\underline{x}} + \sin(\alpha)\cos(\beta)\underline{\underline{y}} + \sin(\beta)\underline{\underline{z}}$$

with (x, y, z) the initial reference mark. For our application, the vector displacement U is written:

$$v = 1.\underline{\underline{x}} + 2.\underline{\underline{y}} + 3.\underline{\underline{z}}$$

For the tablecloth $GEOX$ (plan $(y ; z)$):

$$x_p = \sin(30)\underline{\underline{y}}$$

The principal direction of the tablecloth is $\underline{\underline{y}}$ ($\theta = 90°$). The deformation is written then:

$$\varepsilon = u_y = 2$$

For the tablecloth $GEOY$ (plan $(x ; z)$):

$$x_p = \cos(40)\underline{\underline{x}} + \sin(40)\underline{\underline{z}}$$

The principal direction of the tablecloth thus forms an angle of $40°$ with the plan of the tablecloth. The deformation is written then:

$$\varepsilon = u_x\cos^2(40) + u_z\sin^2(40) = 1.82635$$

For the tablecloth $GEOZ$ (plan $(x ; y)$):

$$x_p = \cos(15).\cos(70)\underline{\underline{i}}\underline{x} + \sin(15)\cos(70)\underline{\underline{i}}\underline{y}$$

The principal direction of the tablecloth thus forms an angle of $15°$ with the plan of the tablecloth. The deformation is written then:

$$\varepsilon = u_x \cos^2(15) + u_y \sin^2(15) = 1.067$$

These three values will be the analytical values of reference for the validation of calculations.

3 Modelings

According to modelings, the objects are with a grid with different elements:

Modeling a: cubic: 1 element HEXA8 with 8 nodes
faces: 1 element QUAD4 with 4 nodes

Modeling b: cubic: 6 elements TETRA4 with 4 nodes
faces: 2 elements TRIA3 with 3 nodes

Modeling C: even modeling that of A with quadratic elements

Modeling D: even modeling that out of B with quadratic elements

3.1 Sizes tested and results of modeling A

Modeling A consists of an element CUB8 for the cube and of an element QUA4 for each face. The behavior is elastic (orders MECA_STATIQUE then STAT_NON_LINE (checking)).
One tests the values given by EPSI_ELGA; EPSI_ELNO, SIEF_ELGA, SIEF_ELNO, SIEF_ELGA, SIGM_ELNO in various points in the principal directions of the tablecloths of reinforcement, obtained respectively with the orders MECA_STATIQUE and STAT_NON_LINE.

For **MECA_STATIQUE** :

Points of integration	EPSI_ELGA (in the principal direction of the tablecloth of reinforcement)			SIEF_ELGA (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA3 – Point 1 (tablecloth GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (tablecloth GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the tablecloth of reinforcement)			SIGM_ELNO (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
NO1 (tablecloth GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

For **STAT_NON_LINE**

Points of integration	EPSI_ELGA (in the principal direction of the tablecloth of reinforcement)			SIEF_ELGA (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA3 – Point 1 (tablecloth GEOY)	1.8264	1.8263	0,003	3.6527E11	3.6527E11	0
MA4 – Point 1 (tablecloth GEOY)	1.8264	1.8263	0,003	3.6527E11	3.6527E11	0
MA5 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the tablecloth of reinforcement)			SIEF_ELNO (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 - NO1 (tablecloth GEOY)	1.8264	1.8263	0,003	3.6527E11	3.6527E11	0
MA3 - NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

The value of total potential energy is also tested starting from calculation STAT_NON_LINE. The analytical solution is calculated starting from all the uniaxial deformations in the grids, starting from the equation:

$$E_{pot} = \frac{1}{2} \int_{element} \varepsilon . A . \varepsilon dv$$

where A indicate the tensor of elasticity.

The got results are the following:

	Code_Aster	Analytical reference	Variation
Total potential energy (J)	2.8173E10	2.8173E10	0

3.2 Sizes tested and results of modeling B

Modeling B consists of six elements TETRA4 for the cube and of two elements TRIA3 for each face.
The behavior is elastic by using the order MECA_STATIQUE then the order STAT_NON_LINE (checking).
One tests the values given by EPSI_ELGA; EPSI_ELNO, SIEF_ELGA, SIEF_ELNO ; SIEF_ELGA; SIGM_ELNO in various points in the principal directions of the tablecloths of reinforcement, obtained respectively with the orders MECA_STATIQUE and STAT_NON_LINE.

For **MECA_STATIQUE** :

Points of integration	EPSI_ELGA (in the principal direction of the tablecloth of reinforcement)			SIEF_ELGA (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA3 – Point 1 (tablecloth GEOZ)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (tablecloth GEOZ)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA11 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA21 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA31 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA41 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA51 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA61 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the tablecloth of reinforcement)			SIGM_ELNO (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 - NO1	1.8263	1.8263	0	3.6527E11	3.6527E11	0

(tablecloth GEOY)						
MA3 - NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA11 - NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA21 - NO1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA31 - NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

For **STAT_NON_LINE**

Points of integration	EPSI_ELGA (in the principal direction of the tablecloth of reinforcement)			SIEF_ELGA (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA3 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA11 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA21 – Point 1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA31 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA41 – Point 1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA51 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA61 – Point 1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the tablecloth of reinforcement)			SIEF_ELNO (in the principal direction of the tablecloth of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0
MA2 - NO1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA3 - NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0
MA11 - NO1 (tablecloth GEOZ)	1,067	1,067	0,001	2.13397E11	2.13397E11	0

MA21 - NO1 (tablecloth GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA31 - NO1 (tablecloth GEOX)	2	2	0	4E11	4E11	0

4 Results of modeling C

Modeling C is identical to modeling A by using quadratic elements (order CREA_MAILLAGE, option LINE_QUAD).

One finds the same results as for modeling A (error compared to the reference solution lower than $0.002\ \%$).

5 Results of modeling D

Modeling D is identical to modeling B by using quadratic elements.

One finds the same results as for modeling D (error compared to the reference solution lower than $0.002\ \%$).

6 Summary of the results and general remarks

The results got for these modelings are identical to the reference solutions. They validate modeling GRILLE_MEMBRANE for four types of different elements in the case of a linear mechanical calculation (MECA_STATIQUE and STAT_NON_LINE).