SSLA100 - Infinite cylinder subjected to a field of voluminal and surface forces

Summary:

This test of linear quasi-static mechanics makes it possible to validate the assignment of a loading of field of forces, surface or voluminal.

The studied structure is cylindrical. The fields with the nodes of voluminal and surface density of forces are read in a file with the Ideas format. For the voluminal loading, the field read varies quadratically according to the distance to the axis; for the surface loading, the field read corresponds to an internal pressure.

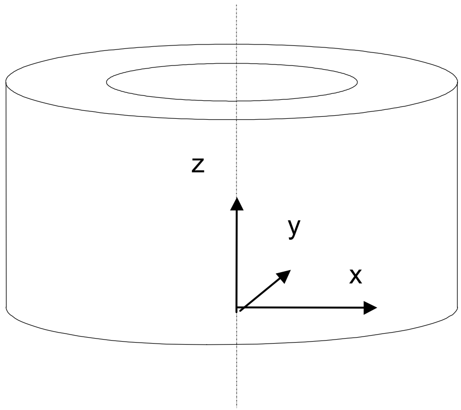
Three modelings of the same problem are carried out:

- modeling 3D;
- axisymmetric modeling 2D;
- modeling 2D plane deformations;

The reference solution is analytical.

1 Problem of reference

1.1 Geometry



Selected geometrical dimensions are the following ones:

- height = $0.5\,m$;
- interior ray = $1\,m$;
- external ray = $1.2\,m$.

1.2 Properties of material

The cylinder consists of a homogeneous material which follows a law of elastic behavior linear:

- $E = 10\,Pa$;
- $\rho = 1\,kg/m^3$;
- $\nu = 0.3$.

1.3 Boundary conditions and loadings (cf [Figure 1.3-a])

The voluminal force considered is radial, it varies in a quadratic way with the ray: $F_V = \alpha . r^2$ with $\alpha = 1\,N/m^3$.
The surface force considered is applied to the internal wall of the cylinder, perpendicular to the wall (is equivalent to an internal pressure imposed on the cylinder): $F_S(r = R_{\text{int}}) = 1\text{N}/m^2$.

The boundary conditions make it possible to be placed on the assumption of the plane deformations on a section of the cylinder: vertical displacements blocked on the sections high and low of the cylinder.

Note:

For modeling 3D, the suppression of the clean modes is ensured by the conditions of the 2D plan applied to the low section of the cylinder. This kind of boundary conditions makes it possible to obtain an axisymmetric solution in displacement, directly comparable to the analytical solution.

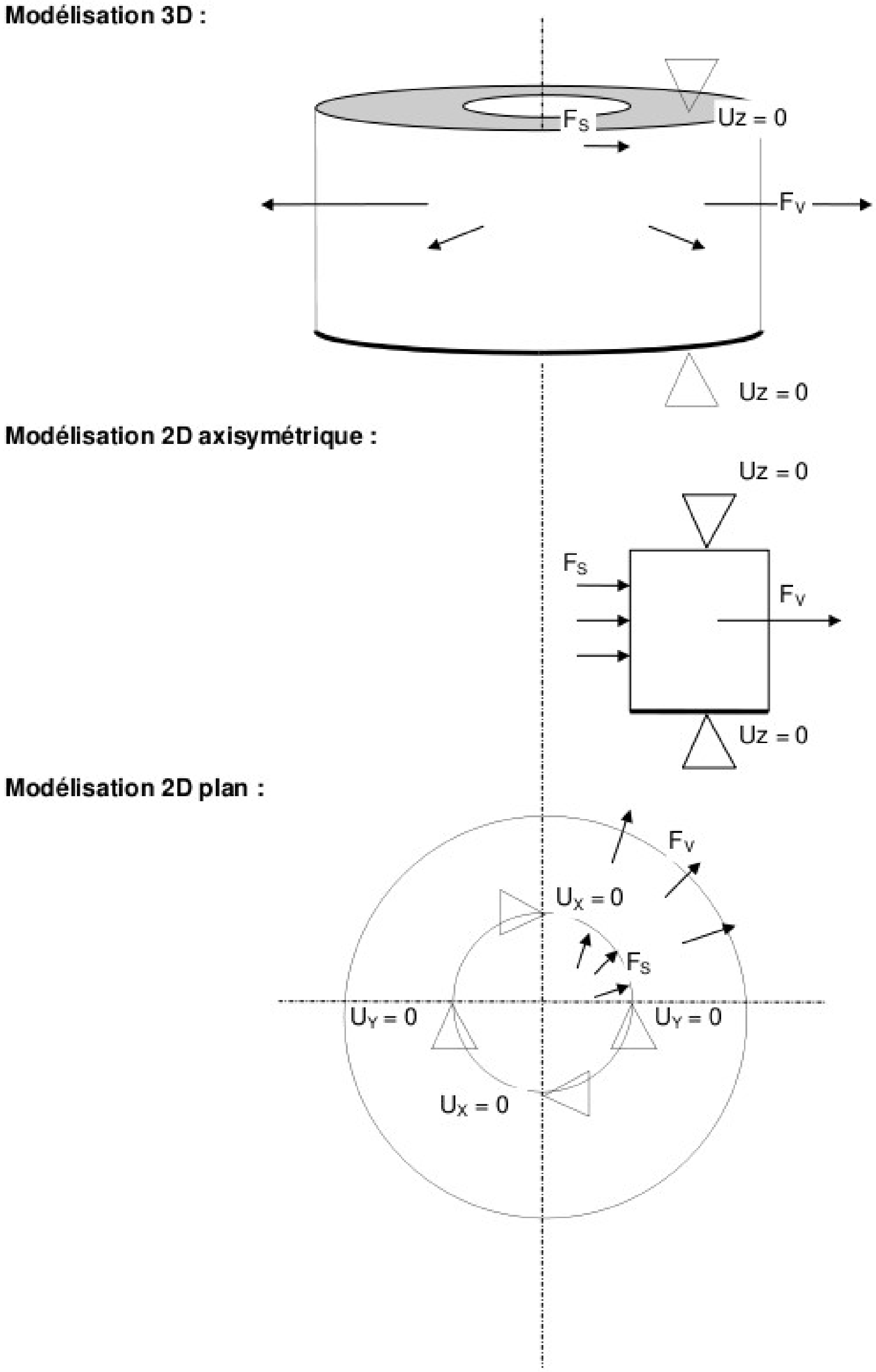


Figure 1.3-a: Boundary conditions and loadings

2 Reference solution

2.1 Method of calculating used for the reference solution

The problem of linear static mechanics axisymmetric considered can be solved in an analytical way. One solves independently the answer to the request forces voluminal and forces surface to summon them then.

Quadratic voluminal force $F_V(r)=\alpha r^2$

One considers the equilibrium equations in cylindrical coordinates:

$$\begin{cases} \dfrac{\partial\sigma_r}{\partial r}+\dfrac{1}{r}\dfrac{\partial\sigma_r}{\partial\theta}+\dfrac{\partial\sigma_{rz}}{\partial z}+\dfrac{\sigma_r-\sigma_\theta}{r}+f_r=0 \\ \dfrac{1}{r}\dfrac{\partial\sigma_{\theta\theta}}{\partial\theta}+\dfrac{\partial\sigma_{\theta z}}{\partial z}+\dfrac{\partial\sigma_{\theta r}}{\partial r}+2\dfrac{\sigma_{r\theta}}{r}+f_\theta=0 \\ \dfrac{\partial\sigma_{zz}}{\partial z}+\dfrac{\partial\sigma_{rz}}{\partial r}+\dfrac{\partial\sigma_{\theta z}}{r\partial\theta}+2\dfrac{\sigma_{r\theta}}{r}+f_z=0 \end{cases}$$

who are simplified being given axial symmetry in: $\dfrac{\partial\sigma_r}{\partial r}+\dfrac{\sigma_r-\sigma_\theta}{r}+f_r=0$

By using the law of behavior then the relations deformation-displacements, one leads to the following differential equation: $u''+\dfrac{u'}{r}-\dfrac{u}{r^2}+\dfrac{f_V}{\dfrac{E(1-\nu)}{(1+\nu)(1-2\nu)}}=0$

The voluminal force applied is of the type: $F_V=\alpha.r^2$

The solution of the differential equation is written then:

$$u=\frac{-c_1}{2r}-\frac{\alpha(1+\nu)(1-2\nu)r^4}{15E(1-\nu)}+c_2 r \quad \text{éq 2.1-1}$$

Two constants of integrations c_1 and c_2 are given thanks to the boundary conditions: $\begin{cases}\sigma(R_{\text{int}})=0\\ \sigma(R_{ext})=0\end{cases}$

One obtains:

$$\begin{cases} c_1=\dfrac{4-3\nu}{1-\nu}\cdot\dfrac{2\alpha}{15}\cdot\dfrac{1+\nu}{E}\cdot\dfrac{R_{\text{int}}^2R_{ext}^2(R_{\text{int}}^3-R_{ext}^3)}{R_{ext}^2-R_{\text{int}}^2} \\ c_2=\dfrac{(1+\nu)(1-2\nu)}{E}\cdot\dfrac{4-3\nu}{1-\nu}\cdot\dfrac{\alpha}{15}\cdot(R_{ext}^3-R_{\text{int}}^2\cdot\dfrac{R_{\text{int}}^3-R_{ext}^3}{R_{ext}^2-R_{\text{int}}^2}) \end{cases}$$

Surface force standard pressure $F_S(R_{\text{int}})=P$

The problem to be solved is of comparable nature, but with a voluminal force applied worthless: $f_V=0$ that is to say $\alpha=0$.

The solution in displacement [éq 2.1-1] is written then: $u = \frac{-c_1}{2r} + c_2 r$, having to observe the conditions: $\begin{cases} \sigma(R_{\text{int}}) = -P \\ \sigma(R_{ext}) = 0 \end{cases}$

What gives:

$$u = P \cdot \frac{1+\nu}{E} \cdot \frac{R_{\text{int}}^2}{R_{ext}^2 - R_{\text{int}}^2} \left(\frac{R_{ext}^2}{r} + (1-2\nu) \cdot r \right) \quad \text{éq 2.1-2}$$

2.2 Results of reference

Digital application:

- height = $0.5\,m$;
- interior ray = $1\,m$;
- external ray = $1.4\,m$;
- E = $10\,Pa$;
- ρ = $1\,kg/m^3$;
- ν = 0.3 ;
- α = $1\,N/m^5$;
- P = $1\,N/m^2$.

by injecting the digital values in the solutions [éq 2.1-1] and [éq 2.1-2] one finds after summation:

$$\begin{cases} u(1.0) = 0.52130982\,m \\ u(1.4) = 0.44203108\,m \end{cases}$$

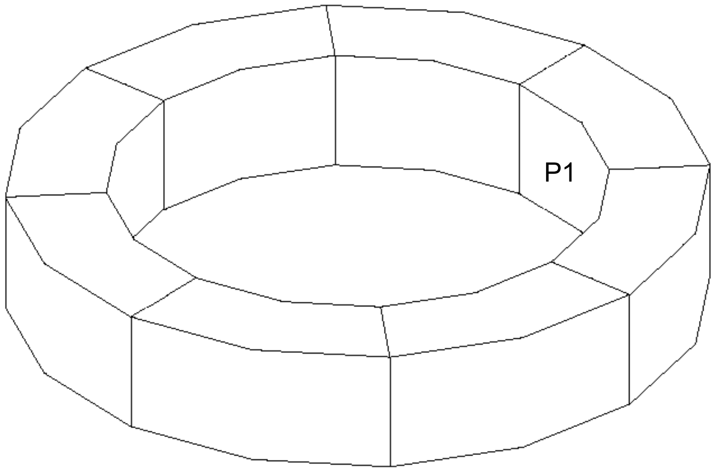
2.3 Uncertainties on the solution

Worthless (analytical reference solution).

3 Modeling A

3.1 Characteristics of modeling

The cylinder is modelled in voluminal elements 3D:



3.2 Characteristics of the grid

The cylinder is represented by a regular grid of quadratic elements with 20 nodes containing:

- 8 elements;
- 96 nodes.

The grid contains 1 only element in the radial and vertical direction and 8 cuttings on the circumference.

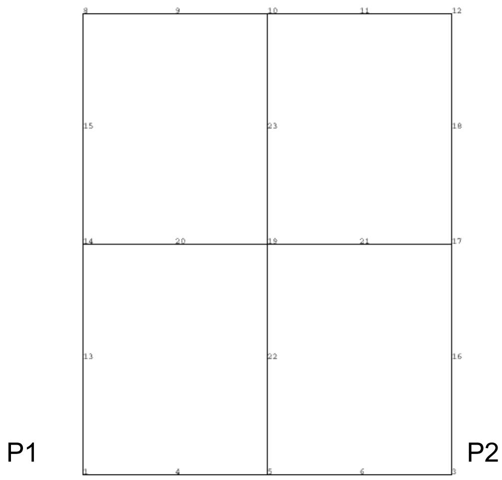
3.3 Values tested

Identification	Moments	Reference
U_X in $P1$	1	0.52130982
U_X in $P2$	1	0.44203108

4 Modeling B

4.1 Characteristics of modeling

A longitudinal section of the cylinder is modelled in voluminal elements 2D, by considering the assumption of axisymetry.



4.2 Characteristics of the grid

The cylinder is represented by a regular grid of quadratic elements with 8 nodes containing:

- 4 elements;
- 21 nodes.

The grid contains 2 cuttings in the radial direction and 2 cuttings in the vertical direction.

4.3 Values tested

Identification	Moments	Reference
U_X in $P1$	1	0.52130982
U_X in $P2$	1	0.44203108

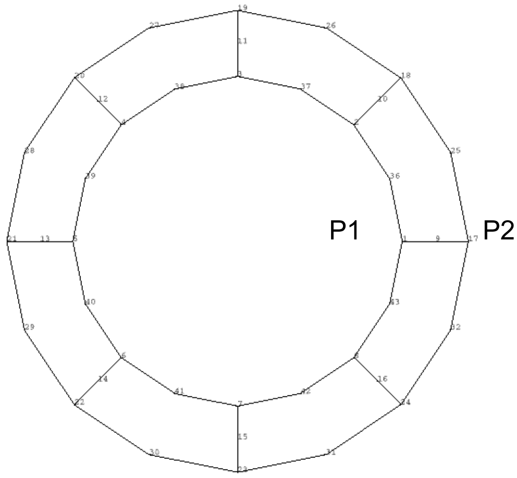
4.4 Notice

Modeling more powerful than the 3D because 2 cuttings in the radial direction and not of circumferential discretization.

5 Modeling C

5.1 Characteristics of modeling

A transverse section of the cylinder is modelled in voluminal elements 2D, by considering the assumption of the plane deformations.



5.2 Characteristics of the grid

The cylinder is represented by a regular grid of quadratic elements with 8 nodes containing:

- 8 elements;
- 40 nodes.

The grid contains 1 only cutting in the radial direction and 8 cuttings in the vertical direction (like the 3D).

5.3 Values tested

Identification	Moments	Reference
U_X in $P1$	1	0.52130982
U_X in $P2$	1	0.44203108

5.4 Remarks

Modeling of performance very close to the 3D because same discretizations circumferential and radial.

6 Summary of the results

Results got by *code_Aster* are very close to the analytical solution, in spite of very coarse networks.

Modelings 3D and 2D plane give further information very close because they present the same discretizations circumferential and radial. Axisymmetric modeling 2D is more powerful because it presents 2 cuttings in the radial direction and not of circumferential discretization.