TPLV100 - Cylinder subjected to conditions with the nonaxisymmetric limits

Summary:

It is about a test in stationary thermics with modeling of Fourier.

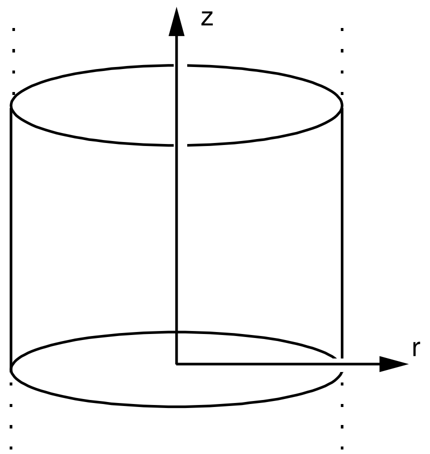
This test validates all the elements of Fourier in thermics (5 different modelings) with various types of boundary conditions: imposed temperature, exchange, imposed flow, source of heat.

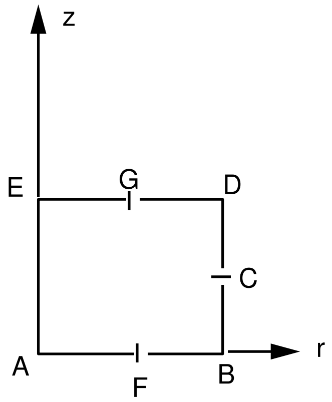
The interest of the test, in addition to the validation of thermics Fourier, lies in the following points:

- comparison between the results and an analytical solution on various harmonics of Fourier (1, 2 and 3),
- homogeneity of the elements between them.

1 Problem of reference

1.1 Geometry





Ray of the cylinder $R=1\ m$.

1.2 Material properties

$\lambda=1\,W/m°C$

1.3 Boundary conditions and loadings

$[EA]$: imposed temperature	$T=T_0=0.°C$
$[BC]$: imposed flow	$\phi=\phi_0=2.\,W/m2°C$
$[CD]$: exchange	$h=2.\,W/m^2°C$ $T_{ext}=2.°C$

2 Reference solution

2.1 Method of calculating used for the reference solution

$$T(r,z,\theta)=R^2\cos l\,\theta$$

with l number of the harmonic of Fourier

$$-\Delta T=(l^2-4)\cos l\,\theta=S$$

$$\vec{\phi}=-(\lambda\vec{\nabla}T)=\begin{bmatrix}-2\mathrm{r}\cos l\,\theta\\0.\\+(lr\sin l\,\theta)\end{bmatrix}$$

on $[AB]$ and $[ED]$: $\phi_0=\vec{\phi}.\vec{n}=0.$

on $[BC]$: $\phi_0=2R=2.$

on $[CD]$: $\vec{\phi}.\vec{n}=2R=\frac{2}{R}(2R^2-R^2)=h(T_{ext}-T)$

from where $h=\frac{2}{R}=2.$

$T_{ext}=2R^2=2.$

Only the source term varies according to the harmonic $(S^l(r,z)=l^2-4)$

In following modelings, one will solve the problem on harmonics 1.2 and 3.

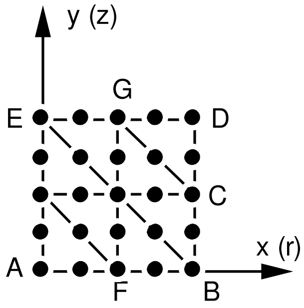
2.2 Results of reference

Temperatures and flow at the points B,C,D,F,G .

3 Modeling A

3.1 Characteristics of modeling

AXIS-FOURIER (TRIA6)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N7
C	1.	0.5	N8
D	1.	1.	N9
E	0.	1.	N3
F	0.5	0.	N4
G	0.5	1.	N6

The axes of description of the grid are $x(r)$ and $y(z)$.

Mode - Fourier: 1 $T(A)=0.$

$S=-3.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.\quad T_{ext}=2.$

3.2 Characteristics of the grid

Many nodes: 25.
Many meshs and types: 8 TRIA6

3.3 Remarks

The number of the mode of Fourier not affecting the loading, the keyword `MODE_FOURIER` is not necessary in the order `CALC_VECT_ELEM`.

The use of the order `CREA_CHAMP/ADZE` is not a recombination of Fourier but a simple validation of this keyword.

3.4 Values tested

	Identification	Reference
$\theta=0$	$T(B)$	1.
	$T(F)$	0.25
	$\phi_r(B)$	– 2
	$\phi_r(F)$	– 1.
	$\phi_\theta(B)$	1.
	$\phi_\theta(F)$	0.5
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.
$\theta=45$	$T(B)$	0.7071
	$T(F)$	0,177
	$\phi_r(B)$	– 1,414
	$\phi_r(F)$	– 0.7071

	$\phi_\theta(B)$	– 0,707
	$\phi_\theta(F)$	– 0.3535
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.
$\theta=135$	$T(B)$	– 0,707
	$T(F)$	– 0,177
	$\phi_r(B)$	1,414
	$\phi_r(F)$	0,707
	$\phi_\theta(B)$	– 0,707
	$\phi_\theta(F)$	– 0.3535
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.

3.5 Remarks

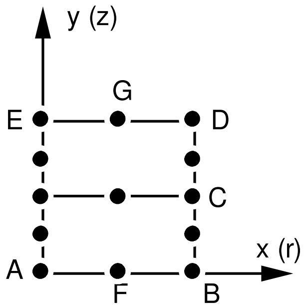
The values of flows to the nodes are realised on the elements containing this node.

It is noticed that the exact solution is not found. This is with the fact that the digital integration of the thermal matrix of rigidity is approximate (formula at 3 points of GAUSS). If a formula at 6 points were used, one would find the solution exactly.

4 Modeling B

4.1 Characteristics of modeling

AXIS_FOURIER (QUAD8)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N4
C	1.	0.5	N5
D	1.	1.	N6
E	0.	1.	N3
F	0.5	0.	N7
G	0.5	1.	N12

The axes of description of the grid are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$
Pas de source term because $S^l(r,z)=0.$ for $l_. =2$

$[BC]$: $\phi=2.$
$[CD]$: $h=2.\quad T_{ext}=2.$

4.2 Characteristics of the grid

Many nodes: 13.
Many meshs and types: 2 QUAD8

4.3 Remarks

The number of the mode of Fourier not affecting the loading, the keyword `MODE_FOURIER` is not necessary in the order `CALC_VECT_ELEM`.

4.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	– 2.
$\phi_r(C)$	– 2.
$\phi_r(D)$	– 2.
$\phi_r(F)$	– 1.
$\phi_r(G)$	– 1.
$\phi_\theta(B)$	2.

$\phi_\theta(C)$	2.
$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

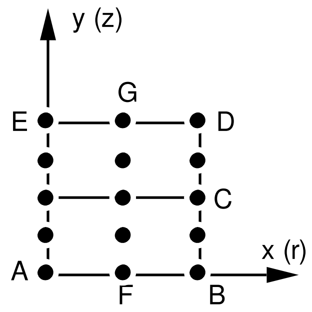
4.5 Remarks

The analytical solution is found exactly.

5 Modeling C

5.1 Characteristics of modeling

AXIS_FOURIER (QUAD9)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N4
C	1.	0.5	N5
D	1.	1.	N6
E	0.	1.	N3
F	0.5	0.	N7
G	0.5	1.	N12

The axes of description of the grid are $x(r)$ and $y(z)$.

Mode - Fourier: 3 $T(A)=0.$

$S=5.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.$ $T_{ext}=2.$

5.2 Characteristics of the grid

Many nodes: 15.
Many meshs and types: 2 QUAD9

5.3 Remarks

The number of the mode of Fourier not affecting the loading, the keyword MODE_FOURIER is not necessary in the order CALC_VECT_ELEM.

5.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	$-2.$
$\phi_r(C)$	$-2.$
$\phi_r(D)$	$-2.$
$\phi_r(F)$	$-1.$
$\phi_r(G)$	$-1.$
$\phi_\theta(B)$	3.
$\phi_\theta(C)$	3.

$\phi_\theta(D)$	3.
$\phi_\theta(F)$	1.5
$\phi_\theta(G)$	1.5
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

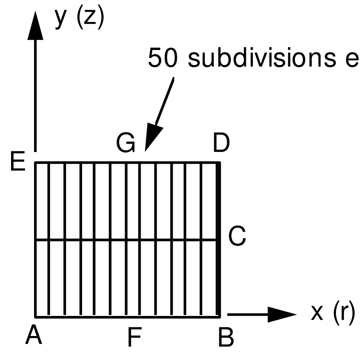
5.5 Remarks

The analytical solution is found exactly.

6 Modeling D

6.1 Characteristics of modeling

`AXIS_FOURIER` (QUAD4)



	r	z	nœud
A	0.	0.	N1
B	1.	0.	N151
C	1.	0.5	N152
D	1.	1.	N153
E	0.	1.	N3
F	0.5	0.	N76
G	0.5	1.	N78

The axes of description of the grid are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$

$S=0.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.$ $T_{ext}=2.$

6.2 Characteristics of the grid

Many nodes: 153
Many meshs and types: 100 QUAD4

6.3 Remarks

The number of the mode of Fourier not affecting the loading, the keyword `MODE_FOURIER` is not necessary in the order `CALC_VECT_ELEM`.

6.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	$-2.$
$\phi_r(C)$	$-2.$
$\phi_r(D)$	$-2.$
$\phi_r(F)$	$-1.$
$\phi_r(G)$	$-1.$
$\phi_\theta(B)$	2.

$\phi_\theta(C)$	2.
$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

6.5 Remarks

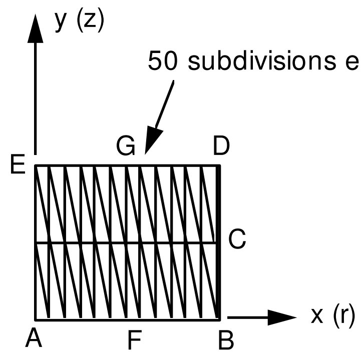
Bad precision recorded on $\phi_r(B)$, $\phi_r(C)$, $\phi_r(D)$ be explained by the fact that B, C and D are nodes of the edge, therefore flows are not realised on adjacent elements in the direction of the variation in temperature (direction R).

This phenomenon is not found on ϕ_θ, because ϕ_θ is balanced by $1/r$.

7 Modeling E

7.1 Characteristics of modeling

AXIS_FOURIER (TRIA3)



	r	z	nœud
A	0.	0.	N1
B	1.	0.	N151
C	1.	0.5	N152
D	1.	1.	N153
E	0.	1.	N3
F	0.5	0.	N76
G	0.5	1.	N78

The axes of description of the grid are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$

$S=0.$ on all the field
$[BC]$: $\phi=2.$
$[CD]$: $h=2.$ $T_{ext}=2.$

7.2 Characteristics of the grid

Many nodes: 153
Many meshs and types: 200 TRIA3

7.3 Remarks

The number of the mode of Fourier not affecting the loading, the keyword MODE_FOURIER is not necessary in the order CALC_VECT_ELEM.

7.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	– 2.
$\phi_r(C)$	– 2.
$\phi_r(D)$	– 2.
$\phi_r(F)$	– 1.
$\phi_r(G)$	– 1.
$\phi_\theta(B)$	2.

$\phi_\theta(C)$	2.
$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

7.5 Remarks

Bad precision recorded on $\phi_r(B)$, $\phi_r(C)$, $\phi_r(D)$ be explained by the fact that B, C and D are nodes of the edge, therefore flows are not realised on adjacent elements in the direction of the variation in temperature (direction R).

This phenomenon is not found on ϕ_θ, because ϕ_θ is balanced by $1/r$.

8 Summary of the results

This problem is correctly solved:

- whatever the number of harmonic of Fourier,
- by the various types of elements (degree 1 or 2).