TTNL101 - Non-linear thermal source in a bar

Summary:

This test checks thermal calculation in the presence of a loading of non-linear source, depend on the temperature.

The reference solution is analytical and variable in time and space. The part considered in three modelings is a symmetrical bar composed of lumpés elements.

The following meshs are checked:

- Modeling a:
  - quadrangles QUAD4, QUAD8, QUAD9 for a modeling AXIS
  - quadrangles QUAD4 for a modeling AXIS_DIAG
- Modeling b:
  - hexahedrons HEXA8 for a modeling 3D_DIAG
- Modeling C:
  - triangles TRIA3 for a modeling AXIS
  - triangles TRIA3 for a modeling AXIS_DIAG

The two ends of the bar are subjected to the conditions of adiabadicity by default. The voluminal source of heat is a linear function of the temperature.

1 Problem of reference

1.1 Geometry

One considers a unidimensional structure (a bar whose side faces are subjected to adiabatic conditions) length $2\mathrm{L}$ occupying the field $[-L; L]$.

The temperature being homogeneous in the normal directions with the bar, calculation can be regarded as 1D.

1.2 Properties of material

$\lambda=2$	thermal conductivity
$\rho C=2$	voluminal heat

1.3 Boundary conditions and loadings

Loading of voluminal source non-linear, function of the temperature:
$s(T)=2-2\times w\times T$ with $w=2$

The boundary conditions are adiabatic on the side faces and of standard worthless temperature imposed at the end of the bar; a condition of symmetry is put in work as regards symmetry (what is equivalent to an adiabatic boundary condition).

The temporal beach $[0.; 1.]$ is discretized in 100 pas de time (lasted of each step of time equalizes with 0.01).

1.4 Initial conditions

An analytical initial state is provided. See the developments in the paragraph detailing the reference solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

The bar is subjected to a source of heat $r(T)=r_0-r_1 T$, where $r_1>0$ for questions of thermal stability. Its initial temperature is worth $T_0(x)$ and the ends of the bar are maintained at a worthless temperature. The evolution of temperature obeys the equation of heat:

$$\rho c\dot{T}=\lambda\nabla^2 T+r(T) \ ; \ T(x,0)=T_0(x) \ ; \ T(-L,t)=T(L,t)=0$$

By standardisation, one can be reduced without loss of general information to the following equation:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+1-\omega^2 u \ ; \ u(x,0)=u_0(x) \ ; \ u(-1,t)=u(1,t)=0$$

To solve this equation, one is interested initially in the asymptotic solution $u_\infty(x)$ who checks:

$$0=\frac{\partial^2 u_\infty}{\partial x^2}+1-\omega^2 u_\infty \ ; \ u_\infty(-1)=u_\infty(1)=0$$

The solution of this linear differential equation of the second order is worth:

$$u_\infty(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh\omega x}{\cosh\omega}\right)$$

The solution of the transitory equation is then obtained by projection of $v=u-u_\infty$ on the clean functions of the Laplacian on $]-1,1[$. To simplify the analysis, an initial condition is adopted u_0 equalize with the first clean mode, namely:

$$u_0(x)=u_\infty(x)-\cos\frac{\pi x}{2}$$

Only the first mode being activated, one is brought back to the solution of a differential equation in first order time, to obtain the solution finally:

$$u(x,t)=u_\infty(x)-\exp\left(-\omega^2 t-\frac{\pi^2}{4}t\right)\cos\frac{\pi x}{2}$$

Lastly, like previously, one goes up u with T by adopting a specific set of parameters, without taking account of the units, so that $T=u$. For that, one takes $\lambda=r_0=\rho c$, $r_1=\omega^2 r_0$, $L=1$ and $T_0(x)=u_0(x)$.

2.2 Results of reference

CAS-test is carried out with $\omega=\sqrt{2}$ and one examines the temperature with $t=1$ in a node of the symmetry plane $(x=0)$. The data are the following ones:

Thermal conductivity	LAMBDA	2.
Voluminal heat-storage capacity	RHO_CP	2.
Initial temperature	T_0	$u_0(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh\omega x}{\cosh\omega}\right)-\cos\frac{\pi x}{2}$ with $\omega=\sqrt{2}$
Source of heat	r_0 r_1	2. 4.

Size tested	$T\ (\ x=0, t=1\ )$
Value of reference	$0.258\,974$

3 Modeling A

3.1 Characteristics of modeling

Modelings are used AXIS then AXIS_DIAG .

Only the half of bar is represented (symmetry).

3.2 Characteristics of the grid

The grid is Co nstitué of 40 of the same quadrangles cuts. following types are considered successively :

Modelings	Meshs
AXIS	QUAD4 QUAD8 QUAD9
AXIS_DIAG	QUAD4

3.3 Largeeurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for a temporal discretization of 100 pas de time.

4 Modeling B

4.1 Characteristics of modeling

Modelings are used 3D_DIAG .

Only the half of bar is represented (symmetry).

4.2 Characteristics of the grid

The grid is Co nstitué of 40 hexaèdr be of even size of type HEXA8 .

4.3 Largeeurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for a temporal discretization of 100 pas de time.

5 Modeling C

5.1 Characteristics of modeling

Modelings are used AXIS then AXIS_DIAG .

Only the half of bar is represented (symmetry).

5.2 Characteristics of the grid

The grid is Co nstitué of 80 triang of the same size. following types are considered successively :

Modelings	Meshs
AXIS	TRIA3 TRIA6
AXIS_DIAG	TRIA3

5.3 Large eurs tested and results

One tests the temperature with $t=1$ in a node of the symmetry plane $(x=0)$

The solution is in conformity with the analytical value with less than 0.1 % for modeling AXIS and of 0.13% for modeling AXIS_DIAG, with a temporal discretization of 100 pas de time.

6 Summary of the results

The results are in conformity with the analytical solution.