FDLV101 - Two cylinders separated by a fluid incompressible

Summary:

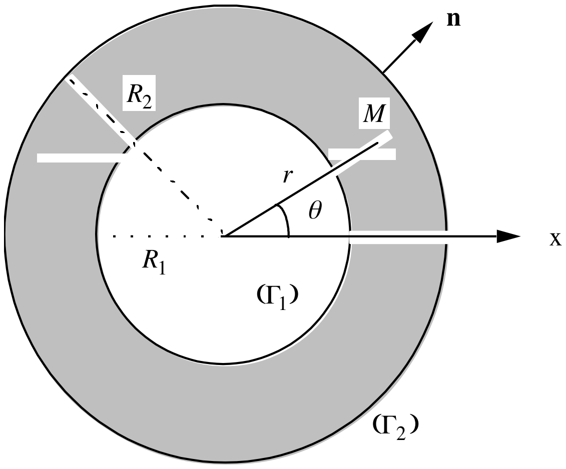
This test of the field of the fluids (fluid coupling/structure) validates the calculation of matrix of added mass if there are several structures immersed in the same fluid.

By a modal analysis, one thus determines the coupled modes of the two structures because of the mass of fluid which separates them. A plane modeling is adopted (thermal for the fluid, and plane deformation for the cylinders).

One finds the modes coupled of the system with less 0.1% analytical result.

1 Problem of reference

1.1 Geometry



Two cylinders separated by incompressible fluid:

interior ray $R_1 = 1.0\,m$ external ray $R_2 = 1.1\,m$

1.2 Material properties

Fluid:

Water: $\rho_0 = 1000.0\,Kg.m^{-3}$

Solid:

Steel: $\rho_S = 7800.0\,Kg.m^{-3}$; $E = 2.\text{E}11\,Pa$; $\nu = 0.3$

Spring connecting the piston to the solid mass:

One places a discrete element on mesh POI1 in the center of the cylinder Γ_1 of stiffness $K1$ and two discrete elements on mesh POI1 on the cylinder Γ_2 on the level of the axis Ox whose stiffness is worth $K2$.

Discrete elements of the type K_T_D_L: $K1 = (1.\text{E}7, 1.\text{E}7, 1.\text{E}7)\,N/m$
$K2 = (5.\text{E}6, 5.\text{E}6, 5.\text{E}6)\,N/m$

1.3 Boundary conditions and loading

One imposes a pressure (i.e. by analogy thermal a worthless temperature [R4.07.03]) in an unspecified node of the fluid.

One imposes a null displacement of the cylinders according to Oy.

1.4 Initial conditions

Without object for the calculation of added mass and the modal analysis.

2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical calculation:

One will suppose that the movements of the cylinders and the fluid are primarily plans. The longitudinal effects will be neglected in front of the transverse effects. The problem is two-dimensional. Taking into account symmetry, the reference mark used is a cylindrical reference mark (r,θ) bound to the central cylinder (see figure above). In this frame of reference and with this particular geometry, the normal derivative $\frac{\partial .}{\partial n}$ is equal to the derivative $\frac{\partial .}{\partial r}$ compared to r.

In all this part, the variable p indicate the hydrodynamic field of pressure in the fluid created by the natural vibrations of the structures, $X_{1/2}$ indicate the clean modes of the cylinder 1 or 2 respectively.

Clean modes of the hulls of border (Γ_1) and (Γ_2) in the absence of fluid are form (n indicate the order of the mode):

$$X_{1n}(r)=\begin{cases}\cos n\theta \quad \text{ou} \quad \sin n\theta \\ 0\end{cases} \text{ and } X_{2n}(r)=\begin{cases}0 \\ \cos n\theta \quad \text{ou} \quad \sin n\theta\end{cases}$$

θ is the azimuth angle. These modes are uncoupled of course. The first component corresponds to the normal displacement of the interior hull, the second with that of the external hull. In fluid volume, there are thus two problems to solve:

$$\Delta p_{1n}=0 \quad \left(\frac{\partial p_{1n}}{\partial n}\right)_{\Gamma_1}=-\rho_f\begin{cases}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{cases} \quad \left(\frac{\partial p_{1n}}{\partial n}\right)_{\Gamma_2}=0 \qquad \text{éq 2.1-1}$$

and:

$$\Delta p_{2n}=0 \quad \left(\frac{\partial p_{2n}}{\partial n}\right)_{\Gamma_1}=0 \quad \left(\frac{\partial p_{2n}}{\partial n}\right)_{\Gamma_2}=-\rho_f\begin{cases}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{cases} \qquad \text{éq 2.1-2}$$

The field p_{1n} corresponds to the field of pressure generated in the fluid if the central hull Γ_1 only vibrate, the field p_{2n} is that created by the external hull Γ_2 if it only vibrates. The linearity of the equation of Laplace makes it possible to solve each problem independently and then to superimpose them to find the field of pressure total.

The solution of the problem [éq 2.1-1] is, in polar coordinates, of the type [bib1]:

$$p_{1n}(r,\theta)=\left\{A r^n+B\left(\frac{1}{r}\right)^n\right\}\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}$$

One must have $n\neq 0$, because if not there is the not-conservation of the volume of the fluid.

Constants A and B are determined by the boundary conditions:

$$\left(\frac{\partial p_{1n}}{\partial n}\right)_{R_1}=-\rho_f\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix} \text{ and } \left(\frac{\partial p_{1n}}{\partial n}\right)_{R_2}=0$$

It is found whereas the field of pressure for each of the two problems is written:

$$p_{1n}(r,\theta)=\frac{\rho_f R_1}{n}\frac{(r/R_1)^n+(R_2/R_1)^n(R_2/r)^n}{(R_2/R_1)^{2n}-1}\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}$$

and:

$$p_{2n}(r,\theta)=\frac{\rho_f R_2}{n}\frac{(R_2/R_1)^n(r/R_1)^n+(R_2/r)^n}{(R_2/R_1)^{2n}-1}\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}$$

Modal coefficients of added mass m^A_{ijnm} are calculated starting from the following formula [R4.07.03] if $i=1$ or 2, $j=1$ or 2, (n,m) belongs to i^2.

$$m^A_{ijnm}=\int_{\Gamma_j} p_{jn} X_{\text{im}}(r).n(\Gamma_j)\,d\Gamma_j$$

The indexing is a little more complex here than in the formula presented in [R4.07.03]: indices I and J refer to the hulls Γ_1 and Γ_2, and indices m and n are associated with the modes of hull. It is noticed that there is coupling of the modes of the various hulls, external and intern.

It is noticed, on the one hand, that the fluid does not couple the modes of indices n different because integrals $\int_\Gamma \cos n\theta\cos m\theta\,d\Gamma$ cancel themselves; in addition, the fluid does not couple either the modes $\cos n\theta$ and $\sin n\theta$ because $\int_{(\Gamma)}\cos n\theta\sin n\theta\,d\Gamma=0$. The only existing coupling is a coupling between the two hulls for the modes of comparable nature.

With each mode n, one associates a matrix of order 4 symmetrical. A submatrix corresponding to projection on the mode n is written:

$$M_1^A=\begin{pmatrix} m^A_{11\text{nn}} & m^A_{12\text{nn}}\\ m^A_{21\text{nn}} & m^A_{22\text{nn}}\end{pmatrix}$$

The total matrix is written: $\begin{bmatrix} M_1^A & 0\\ 0 & M_2^A\end{bmatrix}$ with $M_1^A=M_2^A$

with $m^A_{11\text{nn}}=L R_1\int_0^{2\pi} p_1(R_1,\theta)\begin{Bmatrix}\cos n\theta\\ \text{ou}\\ \sin n\theta\end{Bmatrix}d\theta$

That is to say:

$$m^A_{11\text{nn}}=\frac{\pi}{n}\rho_f R_1^2 L\frac{(R_2/R_1)^{2n}+1}{(R_2/R_1)^{2n}-1} \qquad \text{éq 2.1-3}$$

one will obtain:

$$m^A_{22\mathrm{nn}}=\frac{\pi}{n}\rho_f R_2^2 L\frac{(R_2/R_1)^{2n}+1}{(R_2/R_1)^{2n}-1}$$ éq 2.1-4

and:

$$m^A_{21\mathrm{nn}}=m^A_{12\mathrm{nn}}=-\frac{\pi}{n}\rho_f R_1 R_2 L\frac{2(R_2/R_1)}{(R_2/R_1)^{2n}-1}$$ éq 2.1-5

L indicate here the height of the hulls cylinders in the longitudinal direction.

In our case, only the modes of order are considered $n=1$ hulls: they correspond respectively to the modes of translation of each hull along an axis passing by the center of the central tube: one takes those corresponding to the axis Ox arbitrarily: the coefficients of linear added mass are written:

$$m^A_{11}=\pi\rho_f R_1^2\frac{(R_2/R_1)^2+1}{(R_2/R_1)^2-1}$$

$$m^A_{22}=\pi\rho_f R_2^2\frac{(R_2/R_1)^2+1}{(R_2/R_1)^2-1}$$

$$m^A_{21}=m^A_{12}=-\pi\rho_f R_1 R_2\frac{2(R_2/R_1)}{(R_2/R_1)^2-1}$$

The equation of the generalized movement of the two coupled hulls is written:

$$\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix}+\begin{pmatrix}k_1 & 0\\ 0 & k_2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=-\begin{pmatrix}m_{11} & m_{12}\\ m_{12} & m_{22}\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix}$$

The own pulsations of the coupled system are given by the equation of degree 4:

$$det\left[\begin{pmatrix}m_1+m_{11} & m_{12}\\ m_{12} & m_2+m_{22}\end{pmatrix}\Omega^2-\begin{pmatrix}k_1 & 0\\ 0 & k_2\end{pmatrix}\right]=0$$

Digital application:

$K_1=10^7\,N/m \quad K_2=10^7\,N/m$
$m_{11}=33\,060\,kg/m$
$m_{22}=40\,004\,kg/m$
$m_{12}=-36\,200\,kg/m$

One obtains two Eigen frequencies:

$f_1=1.696\,Hz \quad f_2=4.128\,Hz$

2.2 Results of reference

Analytical

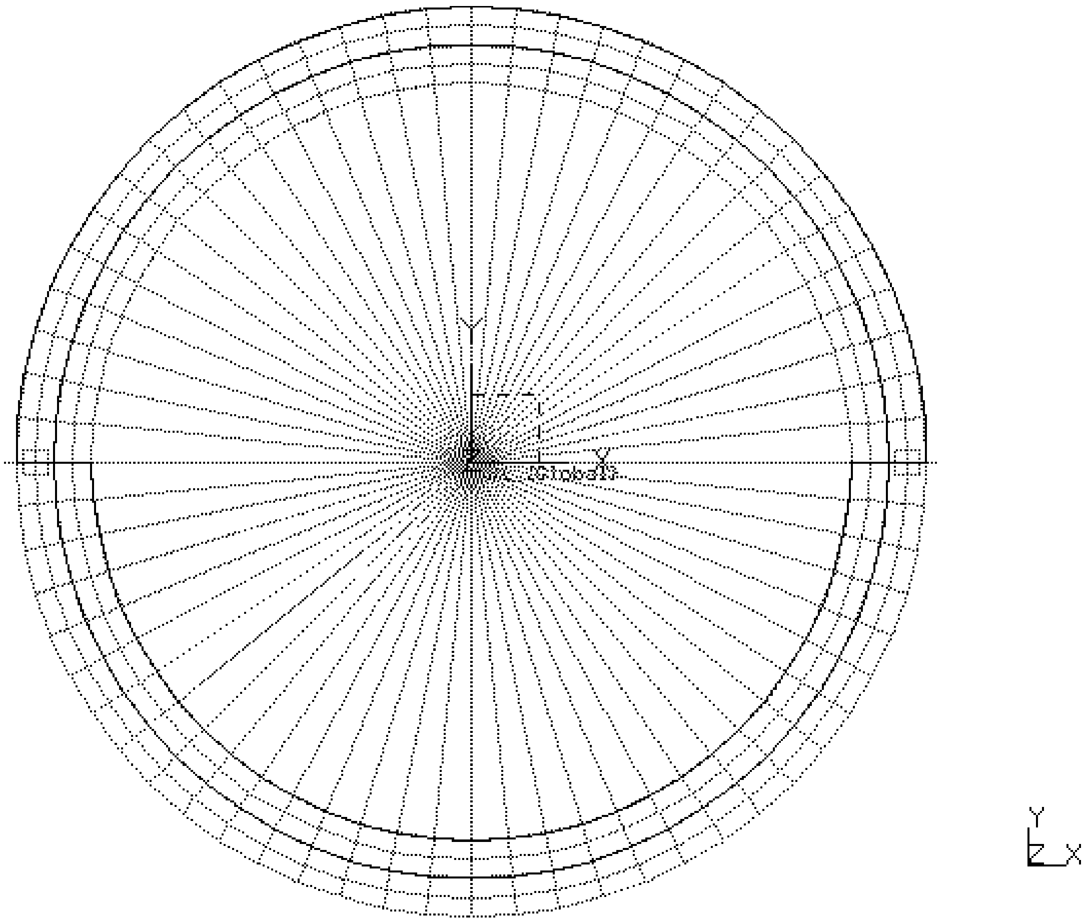
2.3 Bibliographical references

R.J GIBERT. Vibrations of the Structures. Interactions with fluids. Eyrolles (1988).

3 Modeling A

3.1 Characteristics of modeling

Thermal formulation planes for fluid (QUAD4 and SEG2)
Plane and discrete deformation formulation for solid (TRIA3, QUAD4 and POI1)



This modeling is designed to determine the modes of order $n=1$ cylinders. The modes of hulls of a higher nature cannot be simulated by this kind of model, but by a modeling of the type `COQUE_CYL` [U4.22.01].

Cutting =	64 meshs QUAD4 on the circumference of the cylinders 64 meshs TRIA3 on the interior of the interior cylinder 64 meshs SEG2 on the fluid interface/cylinders 2 meshs QUAD4 following the thickness of the fluid 2 meshs QUAD4 following the thickness of the external cylinder

Boundary conditions:	`DDL_IMPO=_F (GROUP_NO= HANGS, DY= 0. )`
	`DDL_IMPO=_F (GROUP_NO= ACCREXT, DY=0. )`
	`TEMP_IMPO=_F (GROUP_NO= TEMPIMPO, TEMP= 0.)`

3.2 Characteristics of the grid

Many nodes: 356 QUAD4

Many meshs and types: 64 TRIA3, 128 SEG2, 3 POI1

3.3 Values tested

Identification	Reference (Hz)	% tolerance
Order of the clean mode i : 1	1,696	0.1%
Order of the clean mode i : 2	4,128	0.1%

3.4 Remarks

Calculations of modes carried out by:

```
CALC_MODES
           OPTION=' PLUS_PETITE',
           CALC_FREQ=_F (NMAX_FREQ=2)
```