FDLV107 - Rigidities added under flow annular

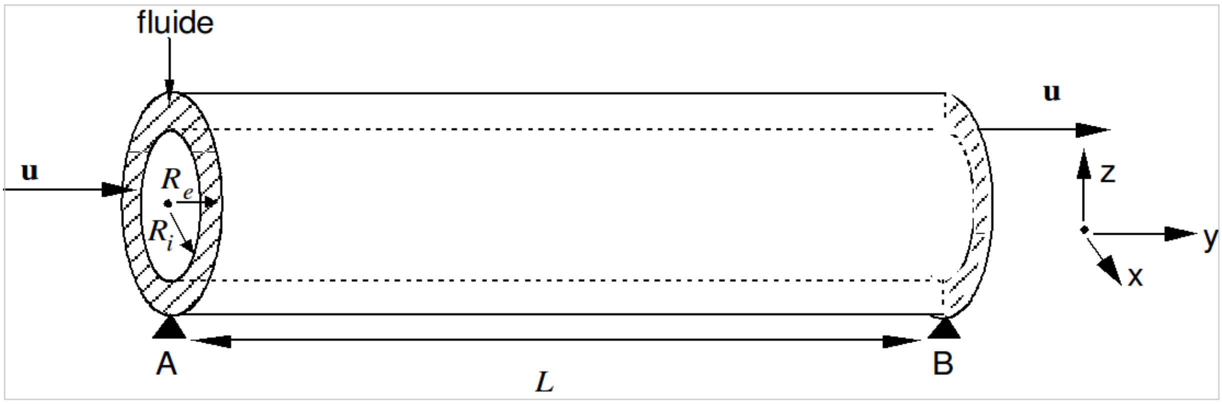
Summary:

This test of the field of the interaction fluid-structure, validates the calculation of rigidity added on a circular cylinder excited on its first mode of inflection kneecap-kneecap and subjected to annular flows various speeds.

One calculates the rigidity added (function speed) on the first mode of inflection of the cylinder. One checks the decrease of the Eigen frequency of the mode, up to zero value for a critical velocity of flow of the fluid.

1 Problem of reference

1.1 Geometry



The system represented on the diagram above is composed of two coaxial cylinders and a fluid flow at the speed U in annular space between the two cylinders. Dimensions are:

- interior ray: $R_i = 1\,m$;
- external ray: $R_e = 1.05\,m$;
- length: $L = 100\,m$.

1.2 Properties of materials

Structure:

Young modulus: $E = 2.10^{11}\,Pa$;
Poisson's ratio: $\nu = 0.3$;
density: $\rho_s = 7800\,kg/m^3$.

Fluid:

density: $\rho = 1000\,kg/m^3$.

1.3 Boundary conditions and loadings

Structure:

blocking of the nodes of the interior cylinder;
kneecap at the points A and B external cylinder.

Fluid:

one imposes various speeds as starter of the fluid field with normal heat fluxes equal to $4\,m/s$, $0.5\,m/s$, $1.5\,m/s$, $2\,m/s$, $2.2\,m/s$ and $2.688\,m/s$ (critical velocity).

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is an approximate analytical solution. The approximate analytical fluctuating potentials to calculate added rigidity are written [bib1]:

$$\begin{cases} \varphi_1(r,\theta,y)= \dfrac{R_e^2}{R_e^2-R_i^2}\left(r+\dfrac{R_i^2}{r}\right) \sin\theta \sin\dfrac{\pi(y+l/2)}{l} \\ \varphi_2(r,\theta,y)= \dfrac{R_e^2}{R_e^2-R_i^2}\left(r+\dfrac{R_i^2}{r}\right) \sin\theta \cos\dfrac{\pi(y+l/2)}{l} \end{cases}$$

The rigidity added on the first mode of inflection of the external cylinder considered as a beam kneecap-kneecap is written [bib1]:

$$K_A=-\frac{\rho}{2}\frac{V_0^2\pi^3 R_e^3}{(R_e^2-R_i^2)\,l}\left(R_e+\frac{R_i^2}{R_e}\right)$$

This rigidity, calculated on a cylindrical geometry, is then assigned to a model with an equivalent degree of freedom.

The system with an equivalent degree of freedom is a system mass-arises equivalent to which one affects a mass equal to the mass of the system increased by the mass added by the fluid and a rigidity equal to the rigidity of the system increased by the rigidity added by the flow for various speeds.

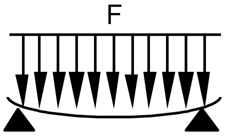
The mass of the system in air is of:

$$M=10292\,kg$$

for an external cylindrical hull thickness:

$$C=2.10^{-3}\,m$$

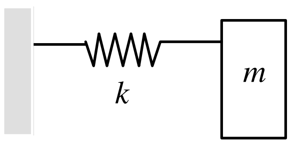
For equivalent rigidity in air of the system “external hull”, one takes the rigidity of a beam subjected to a force distributed over all its length:



$$K=\frac{384EI}{5L^3} \quad \text{avec} \quad I=\frac{\pi d^3 e}{8}=1.649\;10^{-3}\,m$$

thus $K=2.533\,10^4\,N/m$

The equivalent system coupled with the flow is represented by the following diagram:



with $m=M+M_A \quad k=K+K_A$

The own pulsation of the coupled system evolves according to the square rate of flow. If one calls V_{0_c} critical velocity of flow for which rigidity k cancel yourself:

$$\exists V_{0_c}, K+K_A(V_{0_c})=0 \text{ with } V_{0_c}^2=\frac{2(R_e^2-R_i^2)\,l\,K}{\left(R_e+\frac{R_i^2}{R_e}\right)\rho\,\pi^3 R_e^3}$$

then it is shown that:

$$\omega(V_0)=\omega_e(0)\sqrt{1-x^2}$$

where one posed:

$$\omega_e(0)=\sqrt{\frac{K}{M+M_A}}$$ (own pulsation of the system in fluid at rest)

$$x=\frac{V_0}{V_{0_c}}$$ (fallback speed of the flow)

The pulsation of the fluid at rest is worth: $\omega=0.085\,rad/s$.

2.2 Results of reference

One calculates for various rates of flow the Eigen frequency of the system.

$V_0(m/s)$	0.5	1.5	2.	2.2	2,688
$M_A(kg)$	3.486E6	3.486E6	3.486E6	3.486E6	3.486E6
$K_A(N/m)$	– 876.5	– 7888.50	– 14023.95	– 16968.98	– 25330
$M_{total}(kg)$	3.491E+6	=	=	=	=
$K_{total}(N/m)$	24453.5	17441.5	11306.05	8361.00	0.
$f(V_0)\times 10^{-2}(Hz)$	1,318	1,112	0,896	0,772	0.

2.3 Uncertainty on the solution

Semi-analytical solution.

2.4 References bibliographical

1) ROUSSEAU G., LUU H.T. - Mass, damping and stiffness added for a vibrating structure placed in a potential flow - internal Note EDF/DER, HP-61/95/064/A (1995).

3 Modeling A

3.1 Characteristics of modeling

For the geometry on which one evaluates the added coefficients:

Fluid: 1800 elements THER_HEXA8 thermal thermics 1560 elements THER_FACE4 of interface;

Structure: 1200 elements of hull QUAD4 modeling 'DKT'.

For the system with 1 degree of freedom are equivalent: 2 discrete finite elements modeling 'DIS_T'.

3.2 Characteristics of the grid

Grid 1 (hulls cylinders): 1800 meshs HEXA8 1560 meshs QUAD4

Grid 2 (discrete system): 1 mesh SEG2 1 nets POI1

3.3 Values tested

Identification	Reference $\times 10^{-2}$
frequency with $0.5 m/s$	1,318
frequency with $1.5 m/s$	1,112
frequency with $2 m/s$	0,896
frequency with $2.2 m/s$	0,772
frequency with $2.688 m/s$	0.

4 Summary of the results

The variation on the Eigen frequencies increases owing to the fact that when one is close the critical velocity of buckling, the rigidity of the equivalent system must tend towards zero. However, with the errors rounding (since one assigns “to the hand” the values of rigidity added calculated by the operator to a discrete model) do not allow to obtain an own pulsation of the system worthless at the critical velocity.
Variations on the added values of rigidity also remain because the reference solution is built on an semi-analytical solution which leaves the approximation according to which the separation of the variables between the dimension there and the coordinates orthoradiales is possible. It will be noticed that the potentials chosen to describe the disturbance generated by the vibration of the structure in the fluid do not check the equation of Laplace compète but only in one transverse section of the fluid in coordinates orthoradiales. This approximation carried out on the reference solution can explain certain variations with digital calculation.