AHLV101 - Guide of wave at anechoic exit

Summary:

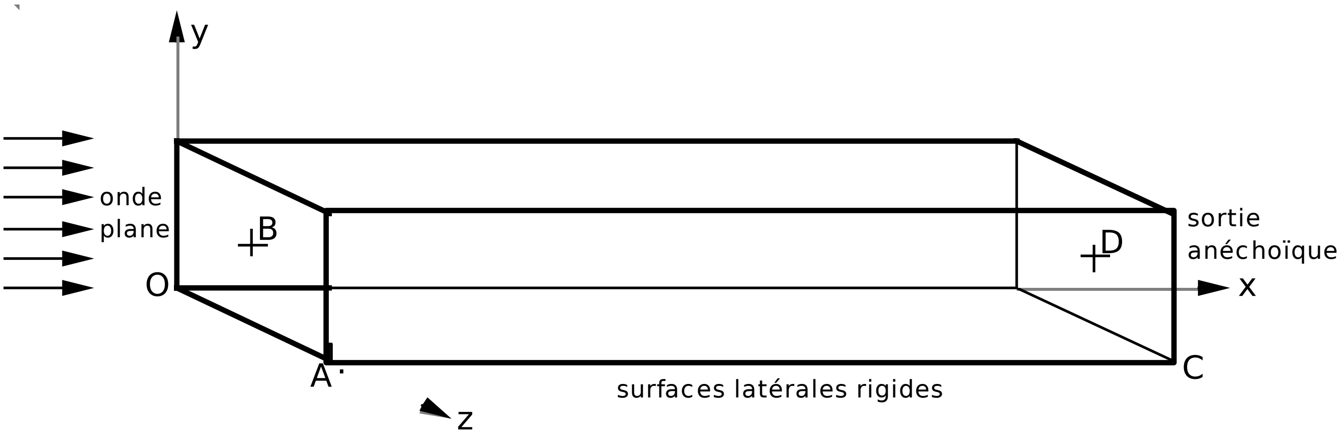
A rectilinear guide of wave at anechoic exit, with rigid walls, whose propagation medium is “normal” air, is excited by a harmonic wave incidental, normal with the face of entry. One calculates the acoustic field of pressure of the harmonic answer by using the élasto-acoustics formulation in pressure - displacement-potential of displacements.

The tests relate to 3 different modelings (finite elements élasto-acoustics three-dimensional, two-dimensional and axisymmetric), they make it possible to validate the matrices of rigidity, mass, impedance and the vector source for 3 modelings.

The result of reference comes from an analytical calculation.

1 Problem of reference

1.1 Geometry



Tube with rectangular section:

length: $L = l_x = 1.0\,m$
height: $h = l_y = 0.1\,m$
width: $l = l_z = 0.2\,m$

Coordinates of the points (in m) :

	A	B	C	D
x	0.	0.	1.00	1.00
y	0.	0.05	0.	0.05
z	0.20	0.10	0.20	0.10

1.2 Properties of materials

Air:

$$\rho = 1.3\,kg.m^{-3}$$
$$c = 343.\,m.s^{-1}$$

1.3 Boundary conditions and loading

Pressure of normal incidental wave at the entry $P_i = P_0 * e^{i\omega t}$ With $P_0 = 1.0\,Pa$

Frequency $f = 500\,Hz$

Impedance at the end CD $Z = \rho c = 445.9\,kg.m^{-2}s^{-1}$

2 Reference solution

2.1 Method of calculating used for the reference solution

The frequencies of the excitation are rather low and jointly the guide of wave is sufficiently long compared to its side dimensions so that one limits oneself to the plane waves: the phenomenon is then identical in all points of a plan of wave, i.e. does not depend on the coordinates describing the points of this plan, y and z for example.

One gives on this assumption the well-known general solution of the equations of acoustics for the two sizes **pressure** p and **acoustic speed** v :

$$v=f\left(t-\frac{x}{c}\right)+g\left(t+\frac{x}{c}\right) \quad \text{éq 2.1-1}$$

$$p=\rho c\left[f\left(t-\frac{x}{c}\right)-g\left(t+\frac{x}{c}\right)\right] \quad \text{éq 2.1-2}$$

The guide is supposed to be closed at the end of X-coordinate L on an impedance Z_L ; it occurs a reflection on the level of this impedance, which gives a wave of return g .

In each point of the guide, there is then superposition of the two functions f and g ; by definition even final impedance Z_L impose on the point of X-coordinate L , enters p and v the relation.

$$\frac{p_L}{v_L}=Z_L$$

In the harmonic case f and g are written:

$$f\left(t-\frac{x}{c}\right)=I\,e^{i\omega\left(t-\frac{x}{c}\right)}$$

$$g\left(t+\frac{x}{c}\right)=R\,e^{i\omega\left(t+\frac{x}{c}\right)}$$

where I and R are determined by the boundary conditions.

In the calculation of the impedance $Z=\frac{p}{v}$ in any point x variable time this time is eliminated, in accordance with the calculation even of the impedances and is written:

$$Z(x)=Z_0\frac{I\,e^{-i\omega\frac{x}{c}}-R\,e^{i\omega\frac{x}{c}}}{I\,e^{-i\omega\frac{x}{c}}+R\,e^{i\omega\frac{x}{c}}}$$

The final impedance becomes:

$$Z_L = Z_0 \frac{I\,e^{-i\omega\frac{L}{c}} - R\,e^{i\omega\frac{L}{c}}}{I\,e^{-i\omega\frac{L}{c}} + R\,e^{i\omega\frac{L}{c}}}$$

One calls $Z_0 = \rho\, c$ iterative impedance.

On the fluid border at the entrance of guide the condition limits of standard incidental wave imposed on $P_i = P_0 e^{i\omega t}$, is obtained by writing at the border the following linear relation:

$$p - \rho\, c\, v_n = P_i \qquad \text{éq 2.1-3}$$

where $v_n = \boldsymbol{v}.\boldsymbol{n}$ is speed according to the unit normal $\boldsymbol{n}$ **outgoing** fluid.

One imposes moreover on the exit of the guide a value of final impedance $Z_L = Z_0$ who makes an anechoic end of it.

The final impedance is equal to the iterative impedance Z_0 when $R=0$, i.e. when there is no wave of return; there is then a wave **progressive** pure in the direction of the incidental wave, that is to say:

$$v = I\,e^{i\omega\left(t - \frac{x}{c}\right)}$$
$$p = \rho\, c\, I\,e^{i\omega\left(t - \frac{x}{c}\right)}$$

thus the relation of imposed incidental wave [éq 2.1-3] is written:

$$p - \rho\, c\, v_n = p(x=0) + \rho\, c\, v(x=0) = 2\,\rho\, c\, I\,e^{i\omega t}$$

from where one identifies $2\,\rho\, c\, I\,e^{i\omega t} = P_i$; one from of deduced the expression from the wave **progressive** of pressure in the guide when one imposes P_i at the entrance of guide:

$$p = \frac{P_i}{2} e^{-i\omega\frac{x}{c}} = \frac{P_0}{2} e^{i\omega\left(t - \frac{x}{c}\right)}$$

2.2 Results of reference

Pressure at the points A , B , C , D (for modelings A, B, C).

2.3 Uncertainty on the solution

Analytical solution.

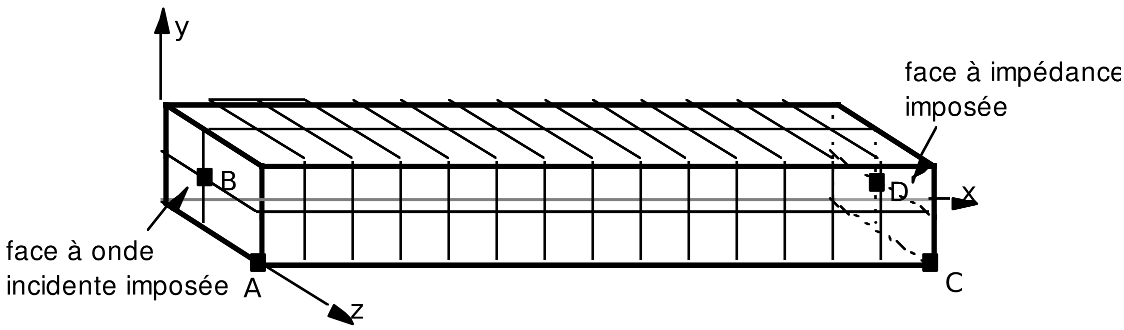
2.4 Bibliographical references

1. F. STIFKENS "Introduction into *Code_Aster* of condition of standard incidental wave into vibroacoustic - Report HP-61/95/026/limits

3 Modeling A

3.1 Characteristics of modeling

Pressure-potential formulation of elements displacements `'3D_FLUIDE'` (`MEFL_HEXA20` and `MEFL_FACE8`)



Cutting =	15	meshs HEXA20 according to the axis of x
	2	meshs HEXA20 according to the axis of y
	2	meshs HEXA20 according to the axis of z

Limiting conditions:

```
ONDE_FLUI:   (GROUP_MA: Entry    NEAR: 1.0  )
IMPE_FACE:   (GROUP_MA: Exit     IMPE: 445.9)
```

Name of the nodes $A=No1$ $B=No780$ $C=No751$ $D=No763$

3.2 Characteristics of the grid

Many nodes: 471
Many meshs and types: 60 HEXA20 8 QUAD8

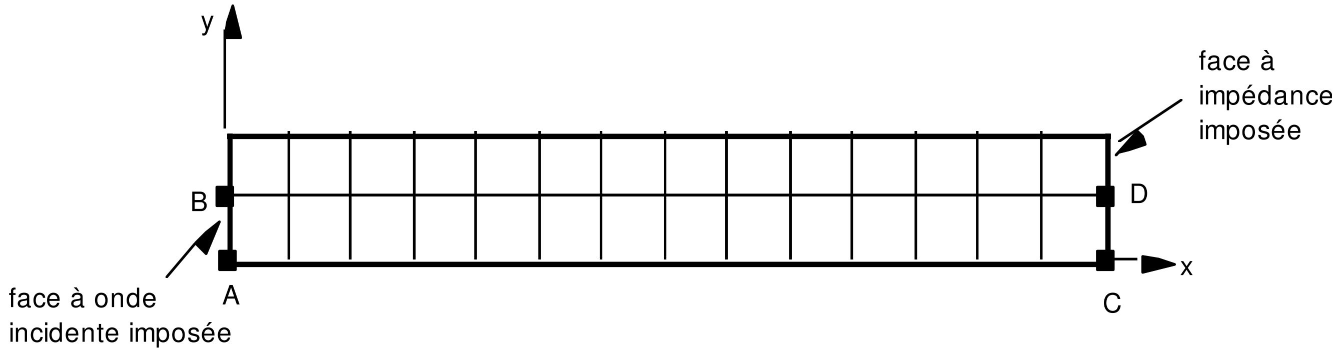
3.3 Values tested

Localization	Sizes	Reference
A	p (reality)	0.5
	p (imag)	0.0
B	p (reality)	0.5
	p (imag)	0.0
C	p (reality)	– 0.482466
	p (imag)	– 0.131252
D	p (reality)	– 0.482466
	p (imag)	– 0.131252

4 Modeling B

4.1 Characteristics of modeling

Formulation pressure potential of elements displacements `'2D_FLUIDE'` (`MEFLSE3` and `MEFLQU8`)



Cutting = 15 meshs QUAD8 according to the axis of x
2 meshs QUAD8 according to the axis of y

Limiting conditions:

```
ONDE_FLUI:   (GROUP_MA: Entry    NEAR: 1.0  )
IMPE_FACE:   (GROUP_MA: Exit     IMPE: 445.9)
```

Name of the nodes $A=No1$ $B=No3$ $C=No751$ $D=No153$

4.2 Characteristics of the grid

Many nodes: 125
Many meshs and types: 30 QUAD8 4 SEG3

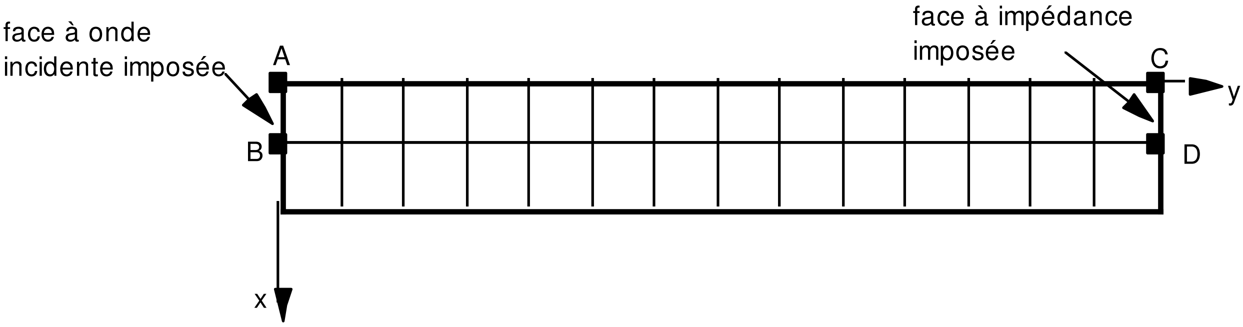
4.3 Values tested

Localization	Sizes	Reference	*Aster*	% difference
A	p (reality)	0.5	0.499997	$6\ 10^{-4}$
	p (imag)	0.0	$1.2\ 10^{-5}$	-
B	p (reality)	0.5	0.499997	$6\ 10^{-4}$
	p (imag)	0.0	$1.2\ 10^{-5}$	-
C	p (reality)	– 0.482466	– 0.482352	$2.4\ 10^{-2}$
	p (imag)	– 0.131252	– 0.131670	$3.2\ 10^{-1}$
D	p (reality)	– 0.482466	– 0.482352	$2.4\ 10^{-2}$
	p (imag)	– 0.131252	– 0.131670	$3.2\ 10^{-1}$

5 Modeling C

5.1 Characteristics of modeling

Pressure-potential formulation of elements displacements `'AXIS_FLUIDE'` (`MEAXFLS3` and `MEAXFLQ8`)



Cutting = 15 meshs QUAD8 according to the axis of y
2 meshs QUAD8 according to the axis of x

Limiting conditions:

```
ONDE_FLUI:   (GROUP_MA: Entry    NEAR: 1.0  )
IMPE_FACE:   (GROUP_MA: Exit     IMPE: 445.9)
```

Name of the nodes $A=No1$ $B=No3$ $C=No151$ $D=No153$

5.2 Characteristics of the grid

Many nodes: 125
Many meshs and types: 30 QUAD8 4 SEG3

5.3 Values tested

Localization	Sizes	Reference	*Aster*	% difference
A	p (reality)	0.5	0.499997	$6\ 10^{-4}$
	p (imag)	0.0	$1.2\ 10^{-5}$	-
B	p (reality)	0.5	0.499997	$6\ 10^{-4}$
	p (imag)	0.0	$1.2\ 10^{-5}$	-
C	p (reality)	– 0.482466	– 0.482352	$2.4\ 10^{-2}$
	p (imag)	– 0.131252	– 0.131670	$3.2\ 10^{-1}$
D	p (reality)	– 0.482466	– 0.482352	$2.4\ 10^{-2}$
	p (imag)	– 0.131252	– 0.131670	$3.2\ 10^{-1}$

6 Summary of the results

The discretization is strong since it is approximately 45 nodes by wavelength. This is why we get results of a high precision: pressure calculated by *Code_Aster* at the least favorable point differs from the theoretical value from less than 1%.

It should be also noted that all modelings used give identical results.